HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop

"Topological and geometrical aspects in complex materials"

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organized by
Valeria Banica (Sorbonne), Radu Ignat (Toulouse), Luc Nguyen (Oxford)

Abstracts

Lia Bronsard (McMaster University)

Core shells and double bubbles in a weighted nonlocal isoperimetric problem

Abstract: We study the Nakazawa–Ohta ternary inhibitory system, which describes domain morphologies in a triblock copolymer as a nonlocal isoperimetric problem for three interacting phase domains. The free energy consists of two parts: the local interface energy measures the total perimeter of the phase boundaries, while a longer-range Coulomb interaction energy reflects the connectivity of the polymer chains and promotes splitting into micro-domains. We consider global minimizers on the two-dimensional torus, in a limit in which two of the species have vanishingly small mass but the interaction strength is correspondingly large. In this limit there is splitting of the masses, and each vanishing component rescales to a minimizer of an isoperimetric problem for clusters in 2D. Depending on the relative strengths of the coefficients of the interaction terms we may see different structures for the global minimizers, ranging from a lattice of isolated simple droplets of each minority species to double-bubbles or core-shells. This represents work with S. Alama, X. Lu, and C. Wang.

Andres Contreras (New Mexico State University)

Local minimizers with unbounded vorticity in 2d Ginzburg–Landau

Abstract: It is known that superconductors possess hysteretic properties that yield vortex configurations with N vortices, for N much different from that present in a global minimizer. In Ginzburg–Landau theory, it is conjectured that local minimizers with prescribed vorticity exist for a range determined by the applied field and the domain. Treating very large vorticities is a challenging problem, and for this reason the best results until recently only cover slowly diverging number of vortices, that is for number of vortices no greater than $|\log \epsilon|$. In joint work with R.L. Jerrard we prove the existence of local minimizers covering almost the whole expected range, for strengths of the applied field of up to a power of $1/\epsilon$. In particular our results show the existence of minimizers with N as large as a power of $1/\epsilon$ for large enough fields.

Raphaël Côte (Université de Strasbourg)

Asymptotic stability of precessing domain walls in ferromagnetic nano wires

Abstract: Ferromagnetic nano wires are materials which attracted sustained attention in physics and industry over the recent years, due to their promising application to data storage. One of the main models to study the evolution of the magnetization on such a material is given by the Landau–Lifschitz–Gilbert equation. It admits remarquable solutions which correspond to topological phase transition, and are called domain walls. I will present recent results on the stability of one or two domains walls, in the presence of a small external magnetic field. These are joint works with Radu Ignat and Guillaume Ferrière.

Georg Dolzmann (Universität Regensburg)

Evolution of vector fields on flexible curves and surfaces

Abstract: In this lecture we discuss some recent progress on a model system consisting of a flexible surface and a vector field defined on the surface in the case in which an interaction between the vector field and the conformation of the surface is present. Recent approaches towards the existence of solutions will be reviewed and short time existence will be established.
The lecture is based on joint work with Christopher Brand (Regensburg), Julia Menzel (Regensburg) and Alessandra Pluda (Pisa).

Ludovic Godard-Cadillac (Université de Strasbourg)

Hölder regularity for collapses of point vortices

Abstract: We study the collapses of point-vortices for the Euler equation in the plane and for surface quasi-geostrophic equations in the general setting of α models. In these models, the kernel of the Biot-Savart law is a power function of exponent $-\alpha$. It is proved that, under a standard non-degeneracy hypothesis, the trajectories of the point-vortices have a Hölder regularity up to the time of collapse. The Hölder exponent obtained is $1/(\alpha+1)$ and this exponent is proved to be optimal for all α by exhibiting an example of a 3-vortex collapse. The same question is then addressed for the Euler point-vortex system in smooth bounded connected domains. It is proved that if a given point-vortex has an accumulation point in the interior of the domain as $t \to T$, then it converges towards this point and displays the same Hölder continuity property. A partial result for point-vortices that collapse with the boundary is also established : we prove that their distance to the boundary is Hölder regular.

Philippe Gravejat (Cergy Paris Université)

Stability of the Ginzburg–Landau vortex

Abstract: We describe the proof of the orbital stability along the flow of the Gross–Pitaevskii equation of the vortex solution (of degree one) for the Ginzburg–Landau equation. This proof is based on the introduction of a functional framework tailored to investigate the minimality and stability properties of this vortex solution. We prove that a renormalized Ginzburg–Landau energy is well-defined in that framework and that the vortex solution is its unique global minimizer up to the invariances by translation and phase shift. We next derive a nonlinear coercivity estimate for this renormalized energy, which eventually leads to the proof of its orbital stability. This is a joint work with Eliot Pacherie (New York University at Abu Dhabi) and Didier Smets (Sorbonne University).

Susana Gutiérrez (University of Birmingham)

Self-similar solutions of the 1d Landau–Lifshitz–Gilbert equation

Abstract: The Landau–Lifshitz–Gilbert equation (LLG) is a continuum model describing the dynamics for the spin in ferromagnetic materials. The main objective of this talk is to present an overview of the construction and study of the dynamical behaviour of self-similar solutions for this model in the one-dimensional case. We will consider both self-similar shrinker and expander solutions.

Matthias Kurzke (University of Nottingham)

Global Jacobians, boundary vortices and polynomials

Abstract: We consider a thin-film limit of ferromagnets that is characterised by the emergence of boundary vortices that can be efficiently described by the global Jacobian, a natural distributional quantity that is well adapted to topological singularities. In some regimes, the model can be reduced to a local PDE with nonlinear boundary conditions. Solutions of this PDE only have degree one singularities, and we prove that there is no clustering of these singularities by reducing this question to properties of certain polynomials.

Xavier Lamy (Université Toulouse III - Paul Sabatier)

On Lebesgue points of entropy solutions to the 2D eikonal equation

Abstract: Entropy solutions of the eikonal equation $|\nabla\varphi| = 1$ in 2D arise as limits of sequences of bounded Aviles–Giga energy. They might be much less regular than viscosity solutions, which have BV gradient, nevertheless their gradients share several structural properties with BV maps, as discovered by De Lellis and Otto. In a joint work with Elio Marconi, we show that non-Lebesgue points have codimension 1 (as they would for BV maps).

Mihai Mariş (Université Toulouse III - Paul Sabatier)

Special classes of solutions to the Gross–Clark system

Abstract: The Gross–Clark system has been used to describe the motion of an uncharged impurity in a Bose condensate. We will present several classes of particular solutions to this system : ground states, bubble-vortices and traveling waves. In each case we will discuss the existence, give some qualitative properties, and compare to corresponding results for the Gross–Pitaevskii equation. This is a joint work with David Chiron and Joe Alhelou.

Benoît Merlet (Université de Lille)

About the non-oriented Aviles–Giga functional

Abstract: We present a non-oriented version of the Aviles–Giga functional which arises as a model for pattern formation (more precisely, for striped patterns in 2D). We show that sequences with uniformly bounded energy as the scale parameter goes to 0 are relatively compact in L^1_{loc}. We also completely describe the limit configurations in the vanishing energy limit case. These results parallel their counterparts for the classical Aviles–Giga functional but new phenomena appear in the non-oriented case and the proofs require new ideas.

Vincent Millot (Université Paris-Est Créteil)

Regularity theory for fractional harmonic maps into spheres

Abstract: In this talk, I will present a regularity theory for minimizing or stationary fractional harmonic maps into spheres underlying the analogies and differences with the classical case. I will also explain their link with classical minimal surfaces and the nonlocal minimal surfaces of Caffarelli-Roquejoffre-Savin.

Antonin Monteil (Université de Paris-Est Créteil)

Magnetic skyrmions confined in a bounded domain

Abstract: Skyrmions are particle-like topological singularities present in some ultrathin ferromagnets under some specific conditions. Their presence can be explained by a specific contribution to the micromagnetic energy, namely the Dzyaloshinskii-Moriya interaction (DMI), which favors rotation of the magnetization vector. We will present a simple model where the presence of a single skyrmion is forced by a degree one condition, and the confinement in a given bounded domain is guaranteed by a constant Dirichlet condition on the boundary. When the DMI strength – tuned by a small parameter – tends to zero, we will see that the energy minimizers concentrate on a point whose position minimizes a renormalized energy that we can compute explicitely for some simple domains. This is joint work with Cyrill B. Muratov (Università Di Pisa), Theresa M. Simon (Universität Münster) and Valeriy V. Slastikov (University of Bristol).

Eliot Pacherie (New York University Abu Dhabi)

Uniqueness of the two vortex travelling wave in the Gross–Pitaevskii equation

Abstract: For the Gross–Pitaevskii equation in dimension 2, it has been shown that there exists a minimizer of the energy at fixed momentum, and that any such minimizer is a travelling wave. For large momentums, they behave like two well separated vortices. In this talk, we will discuss the uniqueness of this minimizer in the large momentum regime, as well as its orbital stability. This is a joint work with David Chiron.

Guanying Peng (Worcester Polytechnic Institute)

Compactness and regularity for a generalized Aviles–Giga functional

Abstract: The classical Aviles–Giga functional is a second order energy functional that models phenomena from thin film blistering to smectic liquid crystals. In this talk we will discuss a generalized Aviles–Giga functional in 2D that was first studied by Bochard and Pegon. The zero set of the potential in this generalized functional is a strictly convex C^1 submanifold with certain symmetry, instead of $\mathbb{S}^1$ in the classical case. Among other things, we generalize the notion of entropies, and use this tool to establish compactness for sequences with bounded energy and regularity for finite energy states. Our regularity result is also closely related to regularity of finite entropy solutions for a class of scalar conservation laws in 1D. This is joint work with Xavier Lamy and Andrew Lorent.

Adriano Pisante (Sapienza Università di Roma)

Torus-like solutions for the Landau–de Gennes model

Abstract: We report on some recent progress (in collaboration with F. Dipasquale and V. Millot) about the study of global minimizers of a continuum Landau–De Gennes energy functional for nematic liquid crystals in three-dimensional domains. First, we discuss absence of singularities for minimizing configurations under norm constraint, as well as absence of the isotropic phase for the unconstrained minimizers, together with the related biaxial escape phenomenon. Then, under suitable assumptions on the topology of the domain and on the Dirichlet boundary condition, we show that smoothness of energy minimizing configurations yields the emergence of nontrivial topological structure in their biaxiality level sets. Then, we discuss the previous properties under both the norm constraint and an axial symmetry constraint, showing that in this case only partial regularity is available, away from a finite set located on the symmetry axis. In addition, we show that singularities may appear due to energy efficiency and we describe precisely the asymptotic profile around singular points. Finally, in an appropriate class of domains and boundary data we obtain qualitative properties of the biaxial surfaces, showing that smooth minimizers exibit torus structure, as predicted in numerical simulations.

Bogdan Raiţă (Scuola Normale Superiore di Pisa)

On concentration effects in pde

Abstract: The aim of this talk is to review old and new results concerning the interaction between nonlinearity and weak convergence of pde constrained sequences. This is a ubiquitous theme in the study of nonlinear pde, of which we will place special emphasis on problems with variational structure. We will review classical results in the study of weak (lower semi)continuity of variational integrals, concerning A-quasiconvexity, compensated compactness, and null Lagrangians. We will conclude with new results pertaining primarily to concentration effects in weak convergence, which we used to answer some open questions. We will discuss connections to Di Perna–Majda measures, defect measures, and concentration compactness. Joint work with A. Guerra, J. Kristensen, and M. Schrecker.

Angkana Rüland (Universität Bonn)

On Rigidity, Flexibility and Scaling Laws: The Tartar Square

Abstract: Highly non-(quasi)-convex, matrix-valued differential inclusions arise in numerous physical applications. One such example is the modelling of shape-memory alloys. In these settings, often the exact differential inclusions display a striking dichotomy between rigidity and flexibility in that
* solutions of sufficiently high regularity obey the “characteristic equations” determined by the differential inclusion, the solutions are rigid in this sense,
* low regularity solutions are highly non-unique and hence extremely flexible.
In order to investigate this dichotomy further, in this talk, I explore the effects of regularizations in the form of singular perturbation problems with vanishing regularization strength for these differential inclusions. Motivated by applications in shape-memory alloys, I discuss the role of scaling properties in the singular perturbation strength. In particular, I discuss the scaling behaviour of a singular perturbation problem for the Tartar square. This talk is based on joint work with Jamie Taylor, Antonio Tribuzio, Christian Zillinger and Barbara Zwicknagl.

Arghir Dani Zărnescu (Basque Center for Applied Mathematics)

Symmetry and multiplicity of solutions in a two-dimensional liquid crystal model

Abstract: We consider a variational two-dimensional Landau-de Gennes model in the theory of nematic liquid crystals in a disk of radius R. We prove that under a symmetric boundary condition carrying a topological defect of degree $\frac{k}{2}$ for some given **even** non-zero integer k, there are exactly two minimizers for all large enough R. We show that the minimizers do not inherit the full symmetry structure of the energy functional and the boundary data. We further show that there are at least five symmetric critical points. This is joint work with Radu Ignat, Luc Nguyen and Valeriy Slastikov.

Barbara Zwicknagl (Humboldt-Universität zu Berlin)

Needle-like martensitic microstructures in shape-memory alloys

Abstract: We consider needle-like microstructures in shape-memory alloys. Such patterns are often observed near macro-interfaces where regions of different laminate orientation meet. Two main quantities describing the geometry of a needle-like domain are its tapering length and the bending angle at the interface. The focus of this talk lies on the tapering length of the needle. We consider typical variational models for such microstructures within the framework of the phenomenological theory of martensite, based on finite or geometrically linearized elasticity. In particular, we discuss recent analytical and numerical results on the scaling of the minimal energy, both in the setting of finite and in linearized elasticity. This talk is based on joint works with S. Conti, M. Lenz, N. Lüthen, M. Rumpf, and J. Verhülsdonk.
