Titles & Abstracts

Speaker: Antti Knowles (University of Geneva)

Title: Euclidean field theories and interacting Bose gases

Abstract: Euclidean field theories have been extensively studied in the mathematical literature since the sixties, motivated by high-energy physics and statistical mechanics. In this course I explain how they arise as high-density limits of interacting Bose gases at positive temperature. I focus on field theories with a nonlocal quartic interaction in dimensions ≤ 3, and with a local quartic interaction in dimension 2. Owing to the singularity of the free field in dimensions higher than one, the interaction is ill-defined and has to be renormalized using infinite mass and energy counterterms. The proof is based on a new functional integral representation of the interacting Bose gas. Based on joint work with Jürg Fröhlich, Benjamin Schlein, and Vedran Sohinger.

References:

- J. Fröhlich, A. Knowles, B. Schlein, and V. Sohinger. The mean-field limit of quantum Bose gases at positive temperature. *J. Amer. Math. Soc.* **35** (2022).
- J. Fröhlich, A. Knowles, B. Schlein, and V. Sohinger . The Euclidean ϕ^4_2 theory as a limit of an interacting Bose gas. To appear in *J. Eur. Math. Soc.* Preprint: arXiv:2201.07632.

Speaker: Sylvia Serfaty (Courant Institute, NYU)

Title: Systems with Coulomb interactions: statistical mechanics and mean-field dynamics

Abstract: We will discuss large systems of particles with Coulomb-type repulsion. The first part of the course will mention the question of mean-field for the dynamics of such systems via a modulated energy approach. The second part will be more expanded and concern the statistical mechanics of such systems (expansion of free energy, LDP for empirical field, fluctuations around the mean-field limit). Several tools and ideas are common to both parts.

Speaker: Fumio Hiroshima (Kyushu University)

Title: Application of FKF for renormalized QFT

Abstract:
1) We show how to construct Feynman-Kac formula (FKF) of various types of Schrödinger semigroups.
2) Applications of FKF to study the ground state of a model in QFT will be introduced.
3) We also show applications of FKF to investigate a renormalized model in QFT.

References:

- F. Hiroshima and O.Matte. Pointwise spatial localization of bound states of the Nelson model. Preprint, 2023.
- B. Hinrichs and O.Matte. Feynman-Kac formula and asymptotic behavior of the minimal energy for the relativistic Nelson model in two spatial dimensions. Preprint: arXiv:2211.14046.

- F. Hiroshima and O.Matte. Ground states and their associated Gibbs measures in the renormalized nelson model, *Rev. Math.Phys.* **34**, 2250002 (2022).
- O. Matte and J. S. Møller. Feynman-Kac formulas for the ultraviolet renormalized Nelson model. *Asterisque* **404** (2018).
- M. Hirokawa, F. Hiroshima and J. Lorinczi. Spin-boson model through a Poisson-driven stochastic process. *Math. Zeitschrift* **277** (2014).
- M. Gubinelli, F. Hiroshima and J. Lorinczi. Ultraviolet renormalization of the Nelson Hamiltonian through functional integration. *J. Funct. Anal.* **267** (2014).
- F. Hiroshima. Functional integral approach to semi-relativistic Pauli-Fierz models, *Adv. in Math.* **259** (2014).

Speaker: Margherita Disertori (University of Bonn)

Title: Supersymmetric method for random matrix problems in quantum diffusion.

Abstract: Supersymmetric approach has proved to be a powerful tool for the study of random matrix models in the context of quantum diffusion, the two prominent examples being the Anderson model and random band matrices. I'll give an introduction to the technique and some recent results.

Speaker: Alessandro Giuliani (Roma Tre University)

Title: Non-trivial fixed point and scaling operators in a 3D fermionic ϕ^4 model

Abstract: After having reviewed the general expected Conformal Field Theory (CFT) nature of the scaling limit of statistical mechanics models at a second order phase transition, we present the construction of the non-trivial infrared fixed point of a 3D fermionic ϕ^4 model with fractional kinetic term, whose scaling dimension is fixed so that the quartic interaction is weakly relevant, in the Renormalization Group (RG) sense. In particular, we prove the analyticity of the anomalous critical exponent of the density correlations, the discrete scale invariance of the field and density correlations and the construction of the corresponding scaling operators, thus providing a direct microscopic foundation of one of the basic CFT axioms. More generally, we discuss the program of proving via constructive RG methods the CFT nature of non-trivial fixed point theories, which the model under investigations appears to be an ideal playground for. Based on joint works with V. Mastropietro, S. Rychkov, G. Scola.

Speaker: Chiara Saffirio (University of Basel)

Title: Stability criteria for the Vlasov-Poisson system and applications to semiclassical problems

Abstract: The Vlasov-Poisson system is a non-linear PDE describing the mean-field time-evolution of particles forming a plasma. In this talk I will present stability criteria for the Vlasov-Poisson equation in strong L^p topologies or in weak topologies (induced by Wasserstein distances), and show how they serve as a guideline to solve semiclassical problems. Different topologies will allow us to treat different classes of quantum states. Based on joint works with J. Chong, L. Lafleche and N. Leopold.

Speaker: Jacob Schach Møller (Aarhus University)

Title: Renormalization of a toy model

Abstract: We investigate a method of ultraviolet renormalization through resolvent resummation, a method going back to J.-P. Eckmann, in the context of a Hamiltonian model for a fermion field interacting with a boson field without particle conservation. We devise a scheme to systematically take into account higher order corrections to the self-energy, thereby extending earlier results of ours to more UV-singular interactions. The talk is based on joint work with Benjamin Alvarez.

Speaker: Benjamin Schlein (University of Zurich)

Title: Upper bounds on the ground state energy of dilute hard spheres.

Abstract: We review some recent estimates on the energy of bosons interacting through a hard-sphere potential. We first discuss Bose gases in the Gross-Pitaevskii regime, in which N hard spheres with radius of order 1/N move on the unit torus; in this setting, we show an upper bound for the ground state energy, valid up to errors that vanish as N tends to infinity. We conclude presenting a simple new bound for hard spheres in the thermodynamic limit, resolving the ground state energy up to an error comparable with the so-called Lee-Huang-Yang corrections.

Speaker: Amanda Young (Technical University of Munich)

Title: Advances on Spectral Gaps Bounds for Quantum Lattice Models

Abstract: A central question in the study of quantum many body systems is to classify models by their phase of matter, and a fundamental quantity in this classification is the existence or non-existence of a spectral gap above the ground state energy. While the importance of the spectral gap is well known, it is a notoriously difficult to rigorously determine the existence of a nonvanishing gap. For frustration-free quantum lattice models, there are only two classes of general methods for proving spectral gaps: methods based on localizing excitations via ground state projections, and finite size criteria.

In this talk, we will review methods from each of these classes and discuss how they were recently applied to prove a bulk gap for a truncated version of the Haldane pseudopotential for the fractional quantum Hall effect. How one approaches proving a spectral gap is often dictated by properties of the ground states. For the truncated Haldane psuedopotential, the ground states are exponentially degenerate in the system size, and to overcome this combinatorial issue one employs a strategy based on identifying invariant subspaces, and this will also be discussed.

This talk is based on joint works with B. Nachtergaele and S. Warzel.
