HAUSDORFF RESEARCH INSTITUTE
FOR MATHEMATICS

UNIVERSITÄT BONN

School on

"K3 surfaces, hyperkähler manifolds, and cubic fourfolds"

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organized by
Sarah Frei (Dartmouth), Richard Haburcak (Dartmouth), Genki Ouchi (Nagoya), Jack Petok (Dartmouth), Xuqiang Qin (UNC)

Abstracts

Chiara Camere (University of Milan)

Finite group actions on hyperkähler manifolds

Abstract: Finite group actions on K3 surfaces and on irreducible compact hyperkähler manifolds have long been studied and are often related to interesting projective families of varieties. In these lectures, I will first review the basic aspects of the theory, including lattice-theoretical tools, constructive methods and deformation theory, and then I will show some interesting examples and applications. Next, I will consider quotients by symplectic group actions, which often are related to new deformation families of (singular) irreducible symplectic varieties, and I will discuss some properties of the family of orbifolds of Nikulin type.

Daniel Huybrechts (University of Bonn)

Brauer groups and twisted sheaves on K3 surfaces

Abstract: This will be a gentle introduction into Brauer groups and twisted sheaves. The emphasis will be on geometric aspects and eventually on moduli spaces of twisted sheaves on K3 surfaces. We will study the different ways to think about Brauer groups as groups of Azumaya algebras, Brauer—Severi varieties, twisted sheaves, $\mathbb{G}_m$-gerbes...How to translate from one to the other, how to define Chern classes, how to split Brauer classes, etc.

Christian Lehn (University of Technology Chemnitz)

Lagrangian Fibrations of Holomorphic Symplectic Varieties

Abstract: By Matsushita's fundamental results, Lagrangian fibrations are essentially the only morphisms on irreducible holomorphic symplectic varieties with positive fiber dimension. We will start by reviewing these results and discuss their validity also for singular symplectic varieties. We will study singular fibers and some of the fundamental conjectures. Towards the end of the course, we will turn to some of the fascinating recent developments in the Hodge theory of Lagrangian fibrations.

Sofia Tirabassi (Stockholm University)

Lieblich–Olsson deformation technique and applications

Abstract: In this lecture series I will explain how one can use deformation theory to study derived categories in positive characteristic.
I will start by giving an overview on what does it mean to "lift" something "to characteristic 0" and when is this possible. Then I will present a baby example: the study of the Fourier–Mukai partners of products of elliptic curves over algebraically closed fields of characteristic at least 5. After that, I will present Lieblich–Olsson deformation technique which allows us to deform derived equivalence. This is a very versatile tools with many applications (not just in positive characteristic!). I will conclude the series by going over some of these applications in greater details.

Pietro Beri (Université Paris Cité)

A geometrical description for birational involutions on an infinite number of Hilbert schemes of points on K3 surfaces

Abstract: Birational automorphisms of Hilbert schemes of points on algebraic K3 surfaces of Picard rank one were recently classified, in a joint work with Al. Cattaneo, in particular, the existence of at most one involution generating the group of birational automorphisms is proven.
This classification does not provide any insight about a geometric description of the automorphisms. Typically, finding such a description is a complex problem: after Beauville's work from 1983, only a finite number of geometric descriptions for automorphisms of Hilbert schemes of points on very general algebraic K3 surfaces have been found. In this talk, I will describe a new infinite family of such involutions and provide some interesting consequences.
This is a joint work in progress with L. Manivel.

Franco Giovenzana (Université Paris-Saclay)

Hyperkähler manifolds from cubic fourfolds

Abstract: In this talk I will present some results, some work in progress and some questions on the geometry of Hyperkähler manifolds constructed starting from cubic fourfolds. The main HK varieties I am interested in are the Fano variety of lines and the LLSvS variety parameterizing flat families of twisted cubics and their flat degenerations. Time permitting, I will touch upon other HK varieties.

Stevell Muller (Saarland University)

On symplectic transformations of $OG10$-type hyperkähler manifolds via cubic fourfolds

Abstract: We know thanks to the work of L. Giovenzana, A. Grossi, C. Onorati and D. Veniani that $OG10$-type hyperkähler manifolds do not admit any non-trivial symplectic automorphisms. What about non-regular symplectic birational transformations? Given a cubic fourfold V, one can construct a hyperkähler manifold X_V of $OG10$-type following a construction of R. Laza, G. Saccà, C. Voisin. Such manifolds are known as LSV manifolds. It can be shown that any symplectic automorphism on V induces a symplectic birational transformation on X_V. In a couple of works with L. Marquand, we study and classify all possible cohomological actions on the $OG10$-lattice which can be realised as symplectic birational transformations. By investigating further the induced action on cohomology, we

exhibit a criterium to decide which of these actions can be realised as induced from a cubic fourfold on an associate LSV manifold.

Sasha Viktorova (KU Leuven)

The defect of a cubic threefold

Abstract: In this talk, we relate the defect $\sigma(X) := b_4(X) - b_2(X)$ of a singular cubic threefold X to various geometric properties of X. The question is motivated by the construction of the exceptional example of a Hyperkähler manifold of type O'Grady 10 from a cubic fourfold by Laza, Saccà and Voisin. By a result of Brosnan, the defect of hyperplane sections of the cubic fourfold is an obstruction for the LSV construction to work. The talk is based on a joint work in progress with Lisa Marquand.
