HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Summer School on

"The Arithmetic of the Langlands Program"

Mai 08 - 12, 2023

organized by
Frank Calegari (Chicago), Ana Caraiani (London), Laurent Fargues (Jussieu), Peter Scholze (Bonn)

Abstracts

George Boxer, Lue Pan, Vincent Pilloni (London, Princeton, Paris)

p-adic Automorphic Forms

Abstract: Lecture 1 (George Boxer) : We discuss the integral theory of p-adic modular forms. We first focus on the modular curve case and describe classical and Higher Hida theory. Then we try to give a conjectural panorama of the theory over general Shimura Varieties.

Lecture 2 (Vincent Pilloni) : We introduce the perfectoid modular curve and the Hodge-Tate period map to projective space. We will consider several classes of equivariant objects over the projective space and study their pull back to the perfectoid modular curve and their descent to finite level modular curves using geometric Sen theory. We give some applications : modular sheaves, Hodge-Tate decomposition, overconvergent modular forms and Higher Coleman theory.

Lecture 3 (Lue Pan) : In this lecture we focus on the locally analytic vectors in completed cohomology. We will identify the arithmetic Sen operator on it with some differential operator constructed geometrically from the Hodge-Tate period map. For applications, we will study the b-cohomology of completed cohomology, explain a classicality result for weight 1 forms, and mention a de Rham version of these results.

Judith Ludwig, Arthur-Cesar le Bras (Heidelberg, Paris)

p-adic Geometry

Abstract: Lecture 1: Adic and perfectoid spaces
The aim of our lectures is to introduce you to the most important concepts in p-adic geometry. We start with a brief recollection of the theory of adic spaces, highlighting key definitions and aspects of the theory (such as Huber and Tate rings and higher rank points), important examples (such as the closed and open unit disc) and limitations (such as sheafyiness questions). We then move on to the theory of perfectoid spaces, define and motivate them, and discuss their most important features.

Lecture 2: Topologies, diamonds and the Fargues—Fontaine curve
We discuss two important topologies on the category of perfectoid spaces, the pro-étale and the v-topology. Both are crucial to introduce and understand the concept of diamonds, which we focus on next. Finally we construct the (adic version of the) Fargues—Fontaine curve, an object of central importance to the arithmetic of the Langlands program, and explain a beautiful formula for it in terms of diamonds.

Lecture 3 and 4: Vector bundles on the Fargues-Fontaine curve and Banach-Colmez spaces (an interesting class of diamonds) are closely related objects. In these two lectures, I'd like to explain this relationship: vector bundles on the curve give rise to Banach-Colmez spaces; conversely, the geometric properties of Banach-Colmez spaces help classifying vector bundles on the curve or studying them in families. This will also be the occasion to make use of some of the p-adic geometry tools discussed in the first two lectures.

Patrick Allen, James Newton (Montreal, Oxford)

Automorphy Lifting

Abstract: Lecture 1 (Patrick Allen): We will discuss what automorphy lifting and R = T theorems are. We will briefly introduce the rings R and T and what it means to produce a map from R to T. We will discuss in particular the example of GL-2(Q) as a warm-up for Lecture 2.

Lecture 2 (James Newton): We will discuss locally symmetric spaces and the relationship between their cohomology and automorphic representations. We will focus on the examples of GL-1 and GL-2 over number fields We will then define Hecke algebras ("T" from lecture 1) in our context, and discuss changing the level at "Taylor-Wiles" primes.

Lecture 3 (Patrick Allen): We will discuss deformation theory in more detail and in particular how presentations of deformation rings are computed by Selmer groups. We will then discuss changing the level at "Taylor-Wiles" primes for deformation rings.

Lecture 4 (James Newton): After giving an introduction to Taylor-Wiles patching, we will explain how it can be used to prove automorphy lifting theorems. The first examples will be with "minimal ramification" conditions. Then we will sketch how Taylor's "Ihara avoidance" method can be used to prove more general results.

Eva Viehmann, Cong Xue (Münster, Paris)

Shtukas

Abstract: Lecture 1: stacks of global shtukas
We start with the motivation and the definition of stacks of global shtukas, with some examples. Then we discuss their local model which are Beilinson-Drinfeld affine grassmanians. Finally we recall the construction of the Hecke correspondances and the action of partial Frobenius morphisms (one of the key properties of stacks of shtukas).

Lecture 2: cohomology of stacks of global shtukas
We start with the definition of the cohomology groups of stacks of global shtukas. Then we discuss their important properties such as the fusion (via the Satake equivalence), the action of the Hecke algebra and the action of the partial Frobenius morphisms. At the end we briefly talk about some

applications of the cohomology.

Lecture 3: We introduce local G-shtukas and moduli spaces parametrizing a given quasi-isogeny class of local G-shtukas. We describe the special fiber of such moduli spaces as an associated affine Deligne-Lusztig variety.

Lecture 4: We discuss relations between moduli spaces of local and global G-shtukas. These are function field analogs of the Rapoport-Zink uniformization theorem and of Mantovan's product formula.

Toby Gee, Xinwen Zhu (London, Standorf)

Categorical Local Langlands

Abstract: In lectures 2 and 3, I will explain the categorical p-adic Langlands correspondence for $GL_2(Q_p)$ in the "Banach" case, and give some idea of how this is expected to generalise (in particular to the case of $GL_2(Q_{p^2})$.
Almost everything I will say will be drawn from my lecture notes with Matthew Emerton and Eugen Hellmann "An introduction to the categorical p-adic Langlands program" from the IHES summer school, and in particular from sections 7.2-7.4, 7.7 and 7.8, with background material from sections 4 and 6.1. (I won't assume that anyone has looked at any of these sections, but they can be referred to for more details on the things that I do say).
The aim will be to state the results precisely but then concentrate on examples. In particular there may not be much time for motivation, and for this it could be worth skimming the (brief!) introduction to the IHES notes above, which attempts to quickly review the Langlands program, the p-adic Langlands program, and the categorical versions.
