HAUSDORFF RESEARCH INSTITUTE
FOR MATHEMATICS

UNIVERSITÄT BONN

"Derived Workshop"

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organized by
Martin Kalck (Graz), Laura Pertusi (Milano), Shizhuo Zhang (Bonn)

Abstracts

Anna Barbieri (Università di Verona)

Moduli spaces of stability conditions and of quadratic differentials

Abstract: The space of Bridgeland stability conditions is a complex manifold attached to a triangulated category D, parametrizing some t-structures of the category. In some cases, when D is constructed from a Ginzburg algebra of a quiver, it is isomorphic to a moduli space of quadratic differentials on a Riemann surface. I will review this correspondence, which is due to Bridgeland-Smith in the simple zeroes case and was extended in [BMQS] to higher order zeroes, to motivate a tentative construction of a smooth compactification of the stability manifold. This is based on joint work (partially in progress) with M.Moeller, Y.Qiu, and J.So.

Arend Bayer (University of Edinburgh)

Non-commutative abelian surfaces and generalised Kummer varieties

Abstract: Polarised abelian surfaces vary in three-dimensional families. In contrast, the derived category of an abelian surface A has a six-dimensional space of deformations; moreover, based on general principles, one should expect to get "algebraic families" of their categories over four-dimensional bases. Generalised Kummer varieties (GKV) are Hyperkaehler varieties arising from moduli spaces of stable sheaves on abelian surfaces. Polarised GKVs have four-dimensional moduli spaces, yet arise from moduli spaces of stable sheaves on abelian surfaces only over three-dimensional subvarieties.
I present a construction that addresses both issues. We construct four-dimensional families of categories that are deformations of $D^b(A)$ over an algebraic space. Moreover, each category admits a Bridgeland stability conditions, and from the associated moduli spaces of stable objects one can obtain every general polarised GKV, for every possible polarisation type of GKVs. Our categories are obtained from $\mathbb{Z}/2$-actions on derived categories of K3 surfaces. This is based on joint work with Alex Perry, Laura Pertusi and Xiaolei Zhao.

Marcello Bernardara (Institut de Mathématiques de Toulouse)

Semiorthogonal decompositions as markings and autoequivalences

Abstract: In this talk, I will introduce the notion of a marking of a triangulated category D as a choice of a semiorthogonal decompositions and show how this notion, together with spherical CY functor F : C → D, can be used to lift autoequivalences from a triangulated category C to D. This gives a categorical framework to Macmullen's construction of special automorphisms of rational surfaces and allows to extend it to some non-commutative examples. All this is a joint work in progress with E. Macrì.

Lev Borisov (Rutgers University)

On categorification of GKZ hypergeometric systems (work in progress)

Abstract: I will talk about a problem of constructing isotrivial families of triangulated categories which underlie the derived equivalences of local toric mirror symmetry. This can be viewed as an attempt to categorify certain GKZ hypergeometric systems.

Igor Burban (University of Paderborn)

Algebraic geometry of the torus model of the fractional quantum Hall effect

Abstract: The experimental discovery of the quantum Hall eff ect is widely considered to be a one of the major events in the condensed matter physics in the second half of the twentieth century. Both experimental and theoretical aspects of this phenomenon still continue to attract an enormous attention.
In 1993 and Keski-Vakkuri and Wen introduced a model for the quantum Hall effect based on multi-layer two-dimensional electron systems satisfying quasi-periodic boundary conditions. Such a model is specified by a choice of a complex torus E and a symmetric positively definite matrix K of size g with integer coefficients.
The space of the corresponding wave functions turns out to be d-dimensional, where d is the determinant of K. I am going to explain a construction of a hermitian holomorphic bundle of rank d on the abelian variety A (which is the g-fold product of the torus E with itself), whose fibres can be identified with the space of wave function of Keski-Vakkuri and Wen. A rigorous construction of this "magnetic bundle" involves the technique of Fourier-Mukai transforms on abelian varieties. This bundle turns out to be simple and semi-homogeneous. Moreover, for special classes of the matrix K, the canonical Chern-Weil connection of the magnetic bundle is shown to be projectively flat. This talk is based on a joint work with Semyon Klevtsov arXiv:2309.04866.

Sebastian Casalaina-Martin (University of Colorado)

Interactions between derived categories and Hodge theory for threefolds

Abstract: I will discuss some results with Jeff Achter and Charles Vial confirming some consequences for threefolds of a conjecture of Orlov, regarding derived categories and Hodge theory. I will also discuss some recent results with Lisa Marquand and Zheng Zhang on the Hodge theory of cubic threefolds with extra automorphisms, as well as how this can be used to study derived equivalences of the automorphism invariant Kuznetsov categories; this is joint work with Xianyu Hu, Xun Lin, Shizhuo Zhang, and Zheng Zhang.

Daniele Faenzi (Université de Bourgogne)

Moduli of bundles in low genus, Coble hypersurfaces and degeneracy loci

Abstract: Coble hypersurfaces enjoy very special properties related to Abelian varieties and moduli of semistable bundles of rank r with trivial determinant on a curve C of genus g, notably when (g,r) equal to (2,3) or (3,2). Using orbital degeneracy loci arising from Vinberg theta-groups and Hecke cycles, we describe moduli of semistable bundles with fixed odd determinant as subvarieties of Grassmannians, again when (g,r) equals (2,3) or (3,2). The geometry of these loci and of their singularities parallels that of Coble hypersurfaces and is related to (almost generic) projective models of K3 surfaces of genus 13 and 19, and to the Hasset divisor C_{18} of cubic fourfolds.
Joint work with Vladimiro Benedetti, Michele Bolognesi, Laurent Manivel.

James Hotchkiss (Columbia University)

Brauer groups, twisted derived categories, and Hodge theory

Abstract: I will discuss some recent developments on a classical question in the theory of Brauer groups, the period-index problem, based on a variety of methods from derived categories, Hodge theory, and enumerative geometry.

Chen Jiang (Fudan University)

Positivity in hyperkähler manifolds via Rozansky–Witten theory

Abstract: For a hyperkähler manifold X of dimension $2n$, Huybrechts showed that there are constants $a_0, a_2, \ldots, a_{2n}$ such that

$$\chi(L) = \sum_{i=0}^{n} \frac{a_{2i}}{(2i)!} q_X(c_1(L))^i$$

for any line bundle L on X, where q_X is the Beauville–Bogomolov–Fujiki quadratic form of X. Here the polynomial $\sum_{i=0}^{n} \frac{a_{2i}}{(2i)!} q^i$ is called the Riemann–Roch polynomial of X. In this talk, I will discuss recent progress on the positivity of coefficients of the Riemann–Roch polynomial and also positivity of Todd classes. Such positivity results follows from a Lefschetz-type decomposition of the root of Todd genus via the Rozansky–Witten theory, following the ideas of Hitchin, Sawon, and Nieper-Wißkirchen.

Johannes Krah (University of Bielefeld)

Phantoms and exceptional collections on rational surfaces

Abstract: A smooth projective rational surface over an algebraically closed field admits a full exceptional collection. We explain how some classification results regarding exceptional collections, previously known for del Pezzo surfaces, can be extended to the blow-up of the projective plane in 9 points in very general position. On the other hand, on the blow-up of 10 points in general position we construct an exceptional collection of maximal length which is not full. This disproves a conjecture of Kuznetsov and a conjecture of Orlov.

Zhiyu Liu (Zhejiang University)

Brill-Noether reconstruction for Fano threefolds and applications

Abstract: There is a natural subcategory of the derived category of a Fano threefold, now called the Kuznetsov component. I will talk about a Brill-Noether reconstruction theorem for a series of Fano threefolds, using Bridgeland moduli spaces in Kuznetsov components. As an application, first I will provide a uniform proof of categorical Torelli theorems for del Pezzo threefolds of degree >1. Then I will give a complete description of the auto-equivalences of Kunetsov components for several Fano threefolds.

Angela Ortega (Humboldt University)

Generic Prym-Torelli Theorem for cyclic coverings of genus 2 curves

Abstract: We consider cyclic unramified coverings of complex smooth curves of genus 2, to which one can associate a Prym variety, defining in this way the Prym map between the corresponding moduli spaces. If the degree of the covering is a Sophie Germain prime number $d \geq 11$, that is, d and $d - 1/2$ are prime, we show that in these cases the Prym map is generically injective.
This is a joint work with Juan Carlos Naranjo and Irene Spelta.

Nebojsa Pavic (Leibniz University Hannover)

Categorical absorption of cyclic quotient singularities

Abstract: We study the derived category of projective varieties with 1/n(1,...,1) singularities. We give sufficient conditions for when derived categories of such varieties admit a semiorthogonal decomposition into two components; a component containing the information of the singularity and a component encoding the smooth information. Moreover, we described the component containing the information of the singularity explicitly as the derived category of a finite-dimensional algebra R and we describe R as the path algebra of a quiver with relations. Finally, we give examples of projective varieties with a 1/n(1,...,1) singularitiy admitting such decompositions. This is joint work with M. Kalck and Y. Kawamata.

David Rydth (KTH)

Weighted blowups, wall-crossing and weak factorization

Abstract: With stack-theoretic weighted blowups, many algorithms, such as resolution of singularities and destackification, become more natural and efficient. I'll explain how this works for wall-crossings and weak factorization.

Evgeny Shinder (University of Sheffield and University of Bonn)

Categorical absorption of singularities and degenerations

Abstract: I will explain under which conditions the derived category of coherent sheaves on a singular projective variety decomposes into a smooth proper part and a finite-dimensional singular part, generated by the so-called P-infinity objects. The emphasis is on varieties with ordinary double points. I will also explain the obstructions for the existence of such decompositions. The talk is based on a joint work with Alexander Kuznetsov.

Paolo Stellari (Università degli Studi di Milano)

Morita theory for schemes: the interplay between dg and weakly approximable triangulated categories

Abstract: A celebrated result by Rickard shows that for two (coherent) rings the property of being derived Morita equivalent can be detected by any of the triangulated categories naturally associated with them (e.g. perfect complexes, bounded or unbounded derived categories of (finitely) generated modules,..). This is the beginning of the so-called Morita theory for schemes. In this talk we provide a significant generalization of Rickard's result beyond the affine setting. The proof involves a new mixture of techniques coming from dg enhancements and the new theory of weakly approximable triangulated categories: It will follow as a nice application of new results that allow us to intrinsically describe subcategories of weakly approximable triangulated categories. Some additional applications to the categories of singularities will be described. This is joint work in progress with A. Canonaco and A. Neeman.

Jenia Tevelev (UMass Amherst)

Categorical aspects of the KSB correspondence II

Abstract: In the classical papers from 1974, Pinkham and Gabriel studied deformations of varieties with a $\mathbb{G}_m$-action and deformations of finite-dimensional associative algebras. They found the first examples of reducible versal deformation spaces: deformations of the cone over a rational normal quartic and deformations of the 4-dimensional algebra $\mathbb{C}[x,y,z]/(x,y,z)^2$. In a joint work with Giancarlo Urzua, we describe a remarkable embedding of the first versal deformation space into the second. Under this embedding, the Artin component maps to deformations to the path algebra of the Kronecker quiver, whereas the $\mathbb{Q}$-Gorenstein component maps to deformations to the 2×2 matrix algebra. In fact, we constructed this embedding for all 2-dimensional cyclic quotient singularities.

Giancarlo Urzúa (Pontificia Universidad Católica de Chile)

Categorical aspects of the KSB correspondence I

Abstract: There will be two talks about the joint work https://arxiv.org/abs/2204.13225 with Jenia Tevelev. In this first talk, I will start by introducing M-resolutions of 2-dimensional cyclic quotient singularities and the Kollár–Shepherd-Barron correspondence. Then I will move to N-resolutions though Hirzebruch-Jung continued fractions. Using MMP on degenerations of surfaces, I will show how N-resolutions can be constructed, alternatively, from M-resolutions via certain braid group action induced by MMP. I will show how this works for Dolgachev surfaces.

Filippo Viviani (Roma Tor Vergata)

On the classification of compactified Jacobians of nodal curves

Abstract: If a smooth curve degenerates to a nodal curve, what are the possible modular degenerations of the Jacobian? I will give a complete answer to this question, based upon some recent results of Pagani-Tommasi.

Michael Wemyss (University of Glasgow)

Derived Deformations of Crepant Curves

Abstract: Motivated by various contraction conjectures, I will describe the full A_∞ structure associated to a general (-3,1)-curve C inside a smooth CY 3-fold. As a corollary, the noncommutative deformation theory of C can be described as a superpotential algebra derived from what we call free necklace polynomials, establishing a suitably interpreted string theory prediction due to Ferrari, Aspinwall-Katz and Curto-Morrison. This is joint work with Gavin Brown.
