# "Birational Workshop"

**November 20 - 24, 2023**

organized by
**Raymond Cheng (Hannover), Mirko Mauri (ISTA), Roberto Svaldi (Milan)**

## Abstracts

### Monday November 20

**Davesh Maulik** (Massachusetts Institute of Technology)

**Perverse filtrations and abelian fibrations**

**Abstract:** Given a proper map $f\colon X \to Y$, the cohomology of $X$ carries an extra structure, known as the perverse filtration, which measures the singularities of $f$. For an abelian fibration, understanding this filtration precisely is related to questions in enumerative geometry, representation theory, and other areas. In this talk, I will discuss this circle of ideas and explain a technique for studying this filtration via the Fourier-Mukai transform on $DCoh(X)$. As an application, we get a new proof of the P=W conjecture for Higgs bundles on a curve. This is joint work with Junliang Shen and Qizheng Yin.

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**Brent Pym** (McGill University)

**A local Torelli theorem for log symplectic manifolds**

**Abstract:** Log symplectic manifolds are generalizations of holomorphic symplectic manifolds, where the symplectic form is allowed to develop mild poles on a hypersurface; they arise often as natural compactifications of phase spaces in mathematical physics. I will describe a local model for the moduli space of log symplectic manifolds, in a neighbourhood of the ones whose degeneracy divisors are normal crossings. As for the local Torelli theorem for compact holomorphic symplectic manifolds, the moduli space is described in terms of the second cohomology, via the cohomology class of the symplectic form. But while in the symplectic case, the moduli space is smooth, in the log symplectic case it is highly singular and reducible, owing to the possibility of deforming the singularities of the hypersurface when the periods of the symplectic form satisfy certain integrality constraints. As an application of our methods, we produce many new irreducible components of the moduli space of log symplectic structures on $\mathbf{P}^n$, where only a handful were previously known. This talk is based on joint work with Mykola Matviichuk and Travis Schedler.

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**Benjamin Bakker** (University of Illinois, Chicago)

**Algebraicity of Shafarevich maps**

**Abstract:** For a normal complex algebraic variety $X$ equipped with a complex representation V of its fundamental group, a Shafarevich map $f\colon X \to Y$ is a map which contracts precisely those algebraic subvarieties on which $V$ has finite monodromy. Such maps were constructed for projective $X$ by Eyssidieux, and recently have been constructed analytically in the quasiprojective case by Brunebarbe and Deng–Yamanoi–Katzarkov, in both cases using techniques from non-abelian Hodge theory. In joint work with Y. Brunebarbe and J. Tsimerman, we show that these maps are algebraic, and that in fact $Y$ is quasiprojective. This is a generalization of the Griffiths conjecture on the quasiprojectivity of images of period maps, and the proof critically uses o-minimal GAGA.

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**Junliang Shen** (Yale University)

**A Fourier transform for abelian fibrations**

**Abstract:** The work of Beauville and Denisgner-Murre endow the cohomology of an abelian scheme a (motivic) decomposition splitting the Leray filtration. This structure, now known as the Beauville decomposition, is induced by the projectors obtained from the Fourier transform. In this talk, I will discuss a theory of Fourier transform for certain abelian fibrations with singular fibers. It extends the Beauville decomposition for abelian schemes and governs the perverse filtration. As applications, in these geometries we confirm the motivic decomposition conjecture of Corti-Hanamura, and obtain the multiplicativity of the perverse filtration and the interaction between the perverse filtration and the tautological classes. If time permits I will discuss the construction of a Lefschetz decomposition compatible with the Fourier transform in the Lagrangian setting. This is a complement to Davesh Maulik's talk, and is based on joint work with Davesh Maulik and Qizheng Yin.

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**Tuesday November 21**

**Sho Tanimoto** (Nagoya University)

**Sections of Fano fibrations over curves**

**Abstract:** Manin's conjecture predicts the asymptotic formula for the counting function of rational points on a smooth Fano variety, and it predicts an explicit asymptotic formula in terms of geometric invariants of the underlying variety. When you count rational points, it is important to exclude some contribution of rational points from an exceptional set so that the asymptotic formula reflects global geometry of the underlying variety. I will discuss applications of the study of exceptional sets to moduli spaces of sections of Fano fibrations, and in particular I will explain how exceptional sets explain pathological components of the moduli space of sections. If time permits, I will discuss the case of del Pezzo fibrations in more details. This is based on joint work with Brian Lehmann and Eric Riedl.

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**Osamu Fujino** (Kyoto University)

**Minimal model program for projective morphisms between complex analytic spaces**

**Abstract:** We discuss the minimal model program for projective morphisms of complex analytic spaces. Roughly speaking, we show that the results obtained by Birkar–Cascini–Hacon–M$^{\mathrm{c}}$Kernan hold true for projective morphisms between complex analytic spaces.

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**Michael Temkin** (Hebrew University of Jerusalem)

**Filling a few holes in the classical resolution of singularities**

**Abstract:** "... in this field, almost everything is already discovered, and all that remains is to fill a few unimportant holes." Philipp von Jolly in his recommendation to Max Planck not to go into physics. Since 2015 I am taking part in a long project (more precisely, a series of projects) with Dan Abramovich and Jarek Włodarczyk on resolution of singularities in characteristic zero - a field which was (and sometimes still is) considered as accomplished up to a few unimportant holes. To our surprise it turned out that there were (and still are) quite a few fundamental things to discover in this classical and thoroughly explored field, and the new discoveries even provide a more conceptual view on what was known before we started our project. It is impossible to compress all results of this journey in one talk, but I will try to outline a unified view on most of our discoveries in these projects. If time permits in the end I will also say a couple of words about our new project in progress with André Belotto - still in characteristic zero...

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**Cécile Gachet** (Humboldt-Universität zu Berlin)

**Orbifold fundamental groups of log Calabi-Yau surface pairs, and the Jordan constant of** $\mathrm{Bir}(\mathbb{P}^2)$**.**

**Abstract:** This talk is a report on joint work with J. Moraga and Z. Liu. Take a pair $(X, D)$, where $X$ is a complex projective normal variety, and $D$ an effective $\mathbb{Q}$-Weil divisor with coefficients in $[0, 1]$: There is a notion of orbifold fundamental group $\pi_1(X, D)$. Note that through a Galois-style correspondence, the normal subgroups of finite index in this group parametrize finite Galois covers of $X$ that branch above components of $D$ with ramification index controlled by the coefficients of said components in $D$, and may branch arbitrarily above the singular locus of $X$. Such a notion of orbifold fundamental groups has been discussed by various people, mostly in the case where the components of D have coefficients strictly smaller than 1, see e.g., L. Braun's paper on klt Fano pairs and B. Claudon - P. Graf - H. Guénancia 's recent work on uniformization of non-positively curved klt pairs.
In this talk, we explain how the orbifold fundamental group of a Calabi-Yau surface pair with log canonical singularities admits a normal subgroup of index at most 7200 that has nilpotency length at most 2 and total rank at most 4. Note that in particular, it is residually finite. Our proof uses previous results on finiteness of the fundamental groups of klt Fano and klt Calabi-Yau surfaces, running MMPs in dimension 2, analysing pairs where $X$ is a Mori fiber space surface onto a curve, analysing pairs where $X$ is toric and has Picard rank one, and a few cases where $X$ is a non-toric Fano surface of Picard rank one. We also explain why some pairs $(X, D)$ have a virtually nilpotent, but not virtually abelian fundamental group, why some pairs $(X_n, D_n)$ have a nilpotent fundamental group that is also virtually abelian, but in which every normal abelian subgroup has index at least n (for $n$ arbitrarily large), and how the (sharp) constant 7200 that appears here is in fact the Jordan constant of the group $3\mathrm{Bir}(\mathbb{P}^2)$, computed by E. Yasinsky.

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**Wednesday November 22**

**Lie Fu** (Université de Strasbourg)

**Hochschild cohomology and deformation theory of Hilbert schemes of points on surfaces.**

**Abstract:** The main goal of this talk is to explain how to compute the Hochschild cohomology groups of Hilbert schemes of points on surfaces. As an application, we deduce various consequences on the deformation theory of the Hilbert schemes, generalizing and reproving the results of Bottacin, Fantechi, and Hitchin. Our method is non-commutative in nature, and works for the symmetric quotient stack

of varieties of arbitrary dimension. If time permits, I would like to promote an amended version of Boissière's conjecture on twisted Hodge numbers of Hilbert schemes. This is a joint work with Pieter Belmans and Andreas Krug (arXiv:2309.06244).

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**Selim Tayou** (Harvard University)

**Title: The non-abelian Hodge locus**

**Abstract:** Classical finiteness results of Arakelov and Parshin state that a fixed quasi-projective curve can only carry finitely many non-isotrivial families of smooth projective curves of fixed genus $g$. These results have been generalized by Faltings and Deligne for polarized variations of Hodge structure of arbitrary weight. In this talk, I will explain a further generalization which only depends on the topology of the base and not the algebraic structure, giving thus a partial answer to a question asked by Deligne. I will then explain an application proving the algebraicity of the non-abelian Hodge locus, partially solving a conjecture of Simpson. The results in this talk are joint work with Philip Engel.

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**Ziquan Yang** (University of Wisconsin, Madison)

**A new case of BSD conjecture and deformation of line bundles**

**Abstract:** I will talk about two results. The first is a new case of the BSD conjecture, contained in a joint work with Hamacher and Zhao. Namely, we prove the conjecture for elliptic curves of height 1 over a global function field of genus 1 under a mild assumption. This is obtained by specializing a more general theorem on the Tate conjecture. The key geometric idea is an application of rigidity properties of the variations of Hodge structures to study deformation of line bundles in positive and mixed characteristic. Then I will talk about a generalization of such deformation results recently obtained with Urbanik. Namely, we show that for a sufficiently big arithmetic family of smooth projective varieties, there is an open dense subscheme of the base over which all line bundles in positive characteristics can be obtained by specializing those in characteristic 0.

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## Thursday November 23

**Tasuki Kinjo** (Kyoto University)

**Derived microlocal geometry and virtual invariants**

**Abstract:** We will introduce a derived geometric generalization of the microlocal sheaf theory, which gives a new perspective on virtual invariants for derived moduli spaces. As an application, we will construct a 3d refinement of the 2d cohomological Hall algebra due to Kapranov—Vasserot. This talk is based on a forthcoming joint work with Adeel Khan.

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**Claire Voisin** (Institut de Mathématiques de Jussieu-Paris Rive Gauche)

**On the smoothing problem for cycles in the Whitney range**

**Abstract:** Borel and Haefliger asked whether the group of cycle classes on a smooth projective variety $X$ is generated by classes of smooth subvarieties (such cycle classes will be said "smoothable"). Outside the Whitney range, that is, when the codimension $c$ of the cycles is not greater than the dimension $d$, there are many counterexamples, the most recent ones being due to Olivier Benoist. In the Whitney

range where $c > d$, it is known that $(c-1)!z$ is smoothable for any cycle $z$ of dimension $d$. Also Hironaka proved that cycles of dimension at most 3 are smoothable.
I study the cycles obtained by pushing-forward products of divisors under a flat projective map from a smooth variety. I show they are smoothable in the Whitney range and I conjecture that any cycle can be constructed this way. I prove that, for any cycle z of dimension $d$, $(d-6)!z$ can be constructed this way, which implies that $(d-6)!z$ is smoothable if $d < c$. In particular, cycles of dimension $d$ at most 7 are smoothable if $d < c$.

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**Eduard Looijenga** (TBC)

**Mapping classes and moduli spaces of K3 manifolds.**

**Abstract:** We report on a program, carried out jointly with Benson Farb. One can ask: given a closed oriented (real) surface, what is the best way to represent a mapping class of that surface by an actual homeomorphism. Thurston essentially answered this question by using Teichmueller theory. The goal here is to emulate this for a K3 manifold S. The role of Teichmueller space is then taken by several moduli spaces of K3 surfaces (some well-known and others perhaps less so). In this talk the focus will on mapping classes of S that act unipotently on its cohomology. The associated moduli are those of K3 surfaces fibered into genus one curves.

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**Gabi Farkas** (Humboldt-Universität zu Berlin)

**Resonance and Koszul modules.**

**Abstract:** Resonance varieties are cohomological invariants that are studied in a variety of topological, combinatorial and geometric contexts. I will discuss their structure in a general algebraic setting and will present a sharp formula for the Hilbert series of the Koszul module associated to the resonance variety in question. Applications to vector bundles on curves, moduli and Kähler groups, respectively hyperplane arrangements will be presented. Based on joint work with Aprodu, Raicu and Suciu.

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**Friday November 24**

**Junpeng Jiao** (Tsinghua University)

**Boundedness of slc degenerations of polarized log Calabi–Yau pairs**

**Abstract:** Given a family of log pairs over a smooth curve whose general fiber is a log Calabi–Yau pair in a fixed bounded family, suppose there exists a divisor on the family whose restriction on a general fiber is ample with bounded volume, we show that, if the total space of the family has relatively trivial log canonical divisor and the special fiber has slc singularities, then every irreducible component of the special fiber is birationally bounded.

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**Cinzia Casagrande** (Università di Torino)

**Fano 4-folds with $b_2 > 12$ are products of surfaces**

**Abstract:** Let $X$ be a smooth, complex Fano 4-fold, and $b_2$ its second Betti number. We will discuss the following result: if $b_2 > 12$, then $X$ is a product of del Pezzo surfaces. The proof relies on a careful study of divisorial elementary contractions $f\colon X \to Y$ such that the image $S$ of the exceptional divisor

is a surface, together with my previous work on Fano 4-folds. In particular, given $f\colon X \to Y$ as above, under suitable assumptions we show that $S$ is a smooth del Pezzo surface with $-K_S$ given by the restriction of $-K_Y$.

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**Stéphane Druel** (Université de Lyon)

**Codimension 1 foliations with projectively flat tangent bundle**

**Abstract:** In this talk, I will discuss codimension 1 regular foliations on complex projective manifolds with numerically projectively flat thangent bundle. Along the way I will explain new results on the normal bundle of a regular codimension 1 foliation.

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