Report on the Hausdorff Trimester Program

Arithmetic and Geometry

January 2 - April 19, 2013

Organizers: L. Dieulefait, G. Faltings, D.R. Heath-Brown,
Yu. I. Manin, B.Z. Moroz, J-P. Wintenberger

Topics

The main theme of the trimestre was the interplay of different methods used
in modern number theory. H. Carayol lectured on his work relating to the
algebraic properties of the Griffiths-Schmid varieties. The second theme was
the recent work of M. Harris, Kai-Wen Lan, R. Taylor, and J. Thorne, who
had constructed Galois representations associated to not necessarily self-dual
automorphic representations. G. Harder lectured on Eisenstein cohomology.
The main themes, relating to counting rational points on algebraic vari-
eties, were local to global principles, Manin's conjecture, developments of
the Hardy-Littlewood method, and the determinant method. O. Lorscheid
and A.L. Smirnov lectured on some aspects of geometry over "the field of one
element". A.N. Parshin and D.V. Osipov lectured on two-dimensional Lang-
lands correspondence. Several other topics were covered during the trimestre
seminar session and at the research conference.

Goals

Our intention was to emphasise the new results and conjectures in arithmetic
geometry, having direct bearing on the classical number-theoretic problems.
The aim of the session on Serre's conjecture was to report on recent works
linked to that conjecture, in particular about Galois representations and au-
tomorphic representations. The aim of the session on counting rational points
was to report on recent works on the existence, frequency and distribution
of rational points on algebraic varieties.

Organization

We ran a seminar series meeting twice per week; the abstracts of the lectures at the seminar are on the HIM website. Two sessions had been organised: on the recently proved Serre conjecture from the 15th of January until the 14th of February (organisers: L. Dieulefait and J-P. Wintenberger) and on counting rational points on algebraic varieties from the 15th of March until the 14th of April (organiser: D.R. Heath-Brown). About twenty lectures were delivered at the final research conference from the 15th until the 19th of April.

Results

The breadth of the subject prevents us from going into any details, concerning the problems studied by the participants of the trimestre; those problems have been discussed in the individual reports. Let us mention, however, that in his recent work "On torsion in the cohomology of locally symmetric varieties" P. Scholze explicitly quotes our trimestre program. The proceedings volume of the trimestre [1] may be regarded as a more detailed report on the program; it contains about twenty papers, a list of the lectures given at our seminar during the session, the programs of the workshop on the Serre conjecture and of the final research conference. One may also consult the preprints submitted to the HIM by some of the participants of our trimestre.

Acknowledgement

We wish to record our gratitude to the Hausdorff Research Institute, and in particular to its director Prof. Dr. W. Lück for the hospitality and financial support we received.

References

[1] Arithmetic and Geometry, Proceedings of the trimestre at the Hausdorff Institute in Bonn, L.V. Dieulefait, G. Faltings, D.R. Heath-Brown, Yu.I. Manin, B.Z.Moroz, and J-P. Wintenberger (editors), LMS Lecture Notes, Cambridge University Press, to appear.