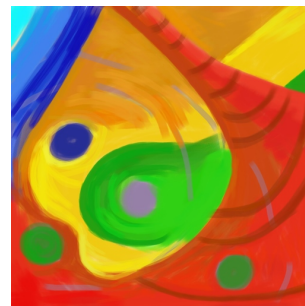


Report on the HIM Trimester Program
**Homotopy theory, manifolds
and field theories**

May 4 – August 21, 2015



Organizers:

Søren Galatius (Stanford University)

Haynes Miller (Massachusetts Institute of Technology)

Stefan Schwede (Universität Bonn)

Peter Teichner (Max Planck Institute for Mathematics, Bonn)

1. TOPICS AND GOALS

Recent years have seen a revitalization and renewed connections between various areas of algebraic topology, in particular homotopy theory, and mathematical approaches to field theories. The aim of this trimester program was to bring together specialists, postdocs and newcomers to spur interactions and to generate new insights. Common tools from homotopy theory and higher category theory played an important role. Some key topics included (∞, n) -categories, factorization homology, algebraic K -theory, moduli spaces of manifolds and equivariant homotopy theory.

2. ORGANIZATION OF THE PROGRAM

The HIM program started with a week long introductory school (May 4-8, 2015), consisting of four lecture series aimed at graduate students and postdocs, introducing some of the main topics of the trimester program. The talks were given by David Ayala on ‘Factorization homology’, by Oscar Randal-Williams on ‘Moduli spaces of manifolds’, by Stefan Schwede on ‘Global equivariant homotopy theory’ and by Peter Teichner on ‘Ring spectra from geometric field theories’.

The next highlight of the program were the Felix Klein Lectures delivered by Charles Rezk on ‘Elliptic cohomology and elliptic curves’ (June 1-19, 2015). The final two lectures were integrated into the main workshop of the program (June 15-19, 2015), which took place at the Max Planck Institute for Mathematics, with 14 exciting talks on many topics of interest to algebraic topologists in general and to the HIM program in particular.

In times without workshops or special events, the program ran a weekly seminar, with two talks every Tuesday, where the participants explained their current research projects. The first slot on May 12 was used as a ‘gong show’ event, where 15 junior participant had the opportunity to a give a short presentation. The participants of the program also started various self-organized working seminars and study groups, on topics such as *Higher categories*

and *Chromatic homotopy theory*, with numerous talks and informal discussions complementing the regular program.

The program closed with another major event, a large *Geometry and Topology* conference (August 17-21, 2015), organized by Wolfgang Lück, Stefan Schwede and Peter Teichner. Although this conference was not formally part of the trimester program, many program participants stayed on an additional week to take advantage of this event.

3. RESULT AND PUBLICATIONS

The trimester program brought a large number of researchers in various different, but adjacent areas of algebraic topology and homotopy theory to Bonn, many for an extended period of several weeks. Many collaborations were thus either initiated by or continued during the trimester program. Some joint projects were deliberately planned (by coordinated visits with coauthors), while others were sparked by informal discussion, with individuals discovering common interests. We list some sample results of cooperations that substantially benefited from the trimester program, chosen from different subareas to reflect the breadth of the program. For a more complete list of joint investigations that turned into research papers we refer to the relevant program web page, see also the link below.

- Lück, Reich, Rognes and Varisco substantially extended their techniques to study topological cyclic homology of group algebras via assembly maps; the results include various split injectivity and rational injectivity results, in particular, for hyperbolic groups and for virtually finitely generated abelian groups.
Reference: W. Lück, H. Reich, J. Rognes, M. Varisco, *Assembly maps for topological cyclic homology of group algebras*, [arXiv:1607.03557](#)
- Mathew, Naumann and Noel took advantage of the trimester program to intensify their collaboration on nilpotence, descent and derived in induction theory in equivariant stable homotopy theory. D. Clausen joined the team during the trimester program, contributing additional expertise in algebraic K -theory.
References: A. Mathew, N. Naumann, J. Noel, *Nilpotence and descent in equivariant stable homotopy theory*, *Adv. Math.* 305 (2017), 994–1084.
D. Clausen, A. Mathew, N. Naumann, J. Noel, *Descent in algebraic K -theory and a conjecture of Ausoni-Rognes*. [arXiv:1606.03328](#)
- Antieau, Gepner and Heller collaborated on the resolution of Schlichting’s conjectures on the negative K -groups of abelian categories. Among other things, they could prove the vanishing of negative K -groups for stable infinity-categories with bounded t -structure and noetherian heart.
Reference: B. Antieau, D. Gepner, J. Heller, *On the theorem of the heart in negative K -theory*. [arXiv:1610.07207](#)
- Riehl and Verity introduce cartesian fibrations, a certain class of ∞ -functors, and their groupoidal variants. These form a cornerstone in the abstract treatment of ‘category-like’ structures a la Street and play an important role in Lurie’s work on quasi-categories. They prove the Yoneda lemma, which has the form of an equivalence between the quasi-category of maps out of a representable fibration and the quasi-category underlying the fiber over its representing element.

Reference: E. Riehl, D. Verity, *Fibrations and Yoneda's lemma in an ∞ -cosmos*. J. Pure Appl. Algebra 221 (2017), 499–564

- Grady and Gwilliam construct a faithful functor from the category of Lie algebroids to the category of L_∞ -spaces. They also show that a shifted-symplectic structure on a dg Lie algebroid produces a shifted-symplectic structure on the associated L_∞ -space. Reference: R. Grady, O. Gwilliam, *Lie algebroids as L_∞ -spaces*. [arXiv:1604.00711](#)
- Antieau and Meier collaborated to compute the Brauer group of the classical elliptic modular stack over a variety of rings. Most of their work is away from the prime 2, but they also show that this group vanishes over the integers. This result in arithmetic algebraic geometry is motivated by questions in chromatic homotopy theory. Reference: B. Antieau, L. Meier, *The Brauer Group of the moduli stack of elliptic curves*. [arXiv:1608.00851](#)
- Greenlees and Meier joined forces during the trimester program to study the Anderson duals of the Real forms of the truncated Brown-Peterson spectra and their localizations (at the prime 2). This calculation establishes Gorenstein duality of specified equivariant shifts for these spectra. Reference: J. P. C. Greenlees, L. Meier, *Gorenstein duality for real spectra*. [arXiv:1607.02332](#), to appear in *Algebr. Geom. Topol.*
- Szymik and Wahl proved that Thompson's group V is acyclic, resolving a conjecture of K. Brown from 1992. The proof involves a homological stability result for Higman-Thompson groups, and a computation of the algebraic K -theory of the category of finitely generated free Cantor algebras. Reference: M. Szymik, N. Wahl: *The homology of the Higman-Thompson groups*. Preprint, 2016, [arXiv:1411.5035v4](#)

At the time of writing this report, 56 preprints had been submitted to the arXiv that were directly related to research initiated or continued during the trimester program. For a complete list that also includes publication information and MathReview identifiers, we refer to the relevant program webpage:

<http://www.him.uni-bonn.de/programs/current-trimester-program/homotopy-theory-2015/publications-preprints/>