

Report on the Trimester Program
Spectral Methods in Algebra, Geometry, and Topology
September - December 2022

Organizers: Paul Balmer, Tobias Barthel, John Greenlees, Henning Krause, Julia Pevtsova

Topics and Goals

The overarching goal of the program was to study the interactions between algebra, geometry, and topology through the lens of tensor-triangular (tt) geometry and its higher variants, as well as to exploit the resulting methods to make progress on longstanding conjectures in these fields. The underlying principle of tt-geometry is that fundamental structures in diverse fields of mathematics can be systematically understood from a uniform point of view. The semester has in particular focused on the following topics though other incarnations of tt-geometry and its applications were also present.

- **Algebra:** The stable module categories of finite groups and finite (super) group schemes, derived categories of finite dimensional algebras, finite tensor categories.
- **Geometry:** Derived categories of quasi-coherent sheaves on schemes and stacks, derived categories of mixed motives.
- **Topology:** The stable homotopy category and its (global) equivariant or motivic analogues; categories of modules over cochain algebras.

Recent years have seen a flurry of activity and groundbreaking progress in each of these areas with the program bringing further advances to each. The specific topics may be organized in the following themes.

1. **Global classification problems:** Classification of thick tensor ideals and localizing tensor ideals as the key approach to capturing the global structure of categories; construction of appropriate support theories.
2. **Local-to-global principles:** Assembly and disassembly in homotopy theory and modular representation theory; adelic techniques in rational equivariant homotopy theory; reconstruction theorems in tt-geometry.

3. **Invariants, duality, and descent:** The computation of Picard groups and higher invariants like Brauer groups; local and global dualities; modern ∞ -categorical descent techniques.

The main goal of the program was to bring together experts in each of the fields above in order to understand the most recent developments, exploit their interactions, and foster future collaborations, while also providing a platform for junior mathematicians to enter this vibrant area of research.

Activities

The program was organized around the following activities.

1. Summer school: *Spectral methods in algebra, geometry, and topology*

Dates: *September 19-23, 2022*

Four mini-courses, consisting of 3 lectures each, were given by

- (a) Dave Benson on *Modular representation theory*;
- (b) Stefan Schwede on *Equivariant homotopy theory*;
- (c) Agnès Beaudry on *Chromatic homotopy theory*;
- (d) Greg Stevenson on *Tensor-triangular geometry*.

They are available on YouTube. At the time of this report, they had garnered over 3'400 views.

2. Workshop: *Spectral methods in equivariant mathematics*

Dates: *October 24-28, 2022*

Organizers: *John Greenlees, Jesper Grodal, Radha Kessar, Henning Krause*

Speakers: Robert Burklund, Natalia Castellana, Ivo Dell'Ambrogio, Martin Gallauer, Paul Goerss, Sira Gratz, Markus Hausmann, Kaif Hilman, Magdalena Kedziorek, Bernhard Keller, Achim Krause, Nick Kuhn, Markus Linckelmann, Nadia Mazza, Amnon Neeman, Luca Pol, Catharina Stroppel, Peter Symonds.

3. Workshop: *Spectra, triangles, and higher structures*

Dates: *December 5-9, 2022*

Organizers: *Paul Balmer, Tobias Barthel, Paul Goerss, Markus Linckelmann, Julia Pevtsova*

Speakers: Tom Bachmann, Scott Balchin, Ben Briggs, Merlin Christ, Pavel Etingof, David Gepner, Jesper Grodal, Jeremiah Heller, Mike Hill, Srikanth Iyengar, Radha Kessar, Janina Letz, Cris Negron, Beren Sanders, Sarah Scherotzke, Vesna Stojanoska, Kent Vashaw, Alexandra Zvonareva. The workshop also included a problem session.

4. Jacob Lurie gave the Felix Klein lectures at the MPI: *A Riemann-Hilbert Correspondence in p-adic Geometry*, Nov 15 - Dec 5, 2022.
5. Several lecture series, including
 - (a) Amnon Neeman on *Triangulated categories via metric techniques*,
 - (b) Jesper Grodal on *Smith theory in algebra, geometry and topology*,
 - (c) Marc Stephan on *Infinity categories*.
6. A weekly seminar on Wednesday afternoon typically consisting of either two one-hour long talks by the participants or a single one-hour long talk and a “gong show”.
 - Sarah Petersen, “Ravenel-Wilson Hopf ring methods in C_2 -equivariant homotopy theory and the $H\mathbb{F}_2$ -homology of C_2 -equivariant Eilenberg-MacLane spaces”
 - Geordie Williamson, “Smith-Treumann theory, geometric representation theory and dreams of K-theory”
 - Mike Hill, “Equivariant approaches to chromatic homotopy”
 - Joseph Ayoub, “Motivic Galois group”
 - Jan Stovicek, “The singularity category of a finite dimensional algebra and finitistic dimensions”
 - Rosanna Laking, “T-structures with Grothendieck hearts in the derived category of a finite-dimensional algebra”
 - Jon Carlson, “Idempotent and Otherwise”
 - Karin Erdmann, “Finite generation of Hochschild cohomology”
 - Neil P Strickland, “Global rational representation theory”
 - Aurélien Djament, “Structure results for generic representations of infinite general linear groups”

- Severin Bartheimer, “Deformations and spectra in noncommutative algebraic geometry”
 - Josh Pollitz, “Cohomological support varieties in local algebra”
 - Yuqing Shi, “Spectral Methods in Unstable Homotopy Theory”
7. “Gong shows”: a series of short talks by the participants with preference given to early-career mathematicians. The goal was to give everyone an opportunity to introduce their research program. We had four installments of those, with more talks scheduled towards the beginning of the program. They were very popular and constructive for initiating interactions and sometimes collaborations.

Results

Short reports collected from the participants at the end of the trimester show enormous enthusiasm for the environment at HIM and for the scheduled activities. They also reveal numerous strong and productive research interactions in a number of networks of collaboration. A great diversity of topics are described, showing progress on both existing projects as well as fresh new endeavors that were initiated at HIM. Finally, it is especially encouraging to see the way that early-career researchers have benefited from coming together, both in what they learned but also in establishing extremely productive networks of research support: this is building a healthy future for our subject.

The trimester resulted in at least 35 publications with more in preparation, classified under Algebraic topology, Category Theory, Representation Theory, Number Theory, K-theory and Homology and Algebraic Geometry on the arxiv, reflecting the breadth of the program. A non-trivial fraction of the publications were new collaborations. Most of these publications can be accessed under the label `HIM-spectral-2022` on the arxiv.

In September 2023 a follow-up workshop *Tensor-Triangular Geometry and Interactions* took place at Oberwolfach, initiated by the same team of organizers with a substantial overlap among the participants. This demonstrates continued interest in the subject and further activities will follow.