Report of the group

Bio-coordination and duality

of the Hausdorff Junior Trimester Program

Stochastic modelling in the life science: From evolution to medicine

Period: 2nd of May – 30th of August

I am **Simone Floreani** (University of Oxford) and I have been the leader of the group *bio-coordination and duality* composed by

- **Adrià Gonzalez Casanova** (UNAM)
- **Nils Hansen** (HU Berlin)
- **Shubhamoy Nandan** (Leiden university)
- **Julio Nava** (UNAM).

The general goal of our group was to explore the use of stochastic duality and graphical constructions in the context of interacting particle systems and population genetics, with the aim of bringing tools from one community to the other one and viceversa. *Stochastic duality* is a useful tool in Markov process theory that allows for studying the expected evolution of certain specific observables of one process via another one, called the dual process, which is hopefully easier. The term *Graphical construction* refers to a space-time representation obtained by drawing vertical lines to denote time intervals, originating from each vertex of the graph where a particle evolves. Additionally, it involves the use of directed or undirected horizontal Poisson arrows to represent particle jumps, connecting adjacent vertices along these vertical lines. To generate the desired stochastic process, it is sufficient to position particles at the beginning of these vertical lines and have them follow deterministic paths defined by these vertical and horizontal segments. Of course, the interpretation of these arrows must be tailored to the specific dynamics of interest.

Project 1

Adrià and I met for the first time during the trimester program and decided to put together our expertise to tackle an important problem arising in non-equilibrium statistical physics: determine the stationary state of certain interacting particle system driven out of equilibrium via particle reservoirs. We focused on one of the particle system that received most of the attention over the last forty years: the symmetric exclusion process (SEP), where symmetric random walks evolving on a connected graph interact via the exclusion rules, which allow only one particle per site. Attempted jumps to occupied sites are suppressed.

When the SEP is studied on a connected finite graph with vertexes in $[N-1] := \{1, \dots, N-1\}$, one easily realizes that there exists a one parameter family of reversible measures given by

$$\mu_{\text{rev}} = \otimes_{x\in[N-1]}\text{Bernoulli}(\rho)$$

with $\rho \in (0,1)$. However if you consider the open symmetric exclusion process, namely the process on the same graph but where now points 1 and $N-1$ are connected, respectively, to a left reservoir and a right reservoir with densities $\rho_L, \rho_R \in (0,1)$, reversibility is broken. Reservoirs are mechanisms that inject and absorb particles imposing a fixed density. In this setting the system has a unique stationary measure called non-equilibrium steady state exhibiting long range correlations. Determining such a measure has received a lot of attention. The so-called Matrix Product Ansatz (MAP) method, invented by Derrida, Evans, Hakim, and Pasquier, makes it possible to compute n-point correlations explicitly when the graph is a segment. However the probabilistic shape of the steady state was not

known. Using stochastic duality, Adrià̀n and I during the trimester program proved that the non-equilibrium steady state of such system is

$$\mu_{stat} = \sum_{I \subset \mathcal{P}([N-1])} F(I) \Big(\otimes_{x \in I} \text{Bernoulli}(\rho_{\text{R}}) \otimes_{\text{y} \in [\text{N}-1] \setminus \text{I}} \text{Bernoulli}(\rho_{\text{L}}) \Big).$$

In the formula above $\mathcal{P}([N-1])$ denotes the power set of $[N-1]$ while the numbers $F(I) > 0$ are such that $\sum_{I \subset \mathcal{P}([N-1])} F(I) = 1$ and given in terms of absorption probabilities of the absorbing stochastic dual process that we built via a graphical construction. Via probabilistic arguments we compute explicitly the factors $F(I)$ when the graph is a homogeneous segment, providing a probabilistic alternative to MAP.

The preprint is available at https://arxiv.org/abs/2307.02481

Project 2

Adrià̀n and Julio initiated and finalized a project entitled *Lookdown construction for a Moran seed-bank model*, where they presented a specific graphical construction, called the lookdown construction, for a Moran seed-bank model with variable active and inactive population sizes and used such construction to derive some connected results on the asymptotic distribution of the time to the most recent common ancestor.

The preprint is available at https://arxiv.org/abs/2305.12489

Other projects and research activities

In June, I invited for one week the researchers Francesco Casini (Modena and Reggio Emilia uni.), Beatriz Salvador (IST Lisbon), Stefan Wagner (LMU Munich) and Hidde van Wiechen (TU Delft). With them I have initiated two projects, one on the hydrodynamic limit of boundary driven interacting particle systems with dormancy and the other about the hydrodynamic limit of interacting active particles in random environment. Stefan Wagner and I made good progresses on a paper in collaboration with Sabine Jansen (LMU Munich) on the algebraic approach to stochastic self-duality on polish spaces and we plan to put it in ArXiv by the end of 2023.

In August, I invited for four days Alberto Chiarini (Padova uni.), with whom I was finalizing a paper on the derivation of the fractional kinetics equation from a Markovian interacting particle system, a paper in collaboration with Frank Redig /TU Delft) and Federico Sau (Trieste Uni.) as well, which appeared on Arxiv in February 2023.

The preprint is available at https://arxiv.org/abs/2302.10156

Events

Adrià̀n and I organized two events. The first one was a one-week workshop entitled *Population Genetics, Interacting Particle Systems and Stochastic Flows: a duality perspective*, with 22 speakers. During the event, Adrià̀n with the invited speakers, Fernando Cordero (Bielefeld uni.) and Jason Schweinsberg (UC San Diego) started to investigate the biological mechanism of adaptation in the presence of seed bank and random environment. The second one was a 3-day research event on random geometries and interacting processes, with 9 speakers.

Conclusions

I attended all the workshops of the trimester program, meeting many new researchers and initiating new collaborations. Overall, I enjoyed a lot participating to the program, learning many new things and starting new projects. Through my permanence at HIM, I received constant support from Silke Steinert-Berndt: the success of organizing workshops and my overall enjoyable experience during my stay can be attributed primarily to her assistance.