Report on the Trimester Program

Interactions between Geometric measure theory, Singular integrals, and PDE

January - April 2022

Organizers: Marianna Csörnyei (Chicago), Tuomas Orponen (Helsinki), Xavier Tolsa (Barcelona), Tatiana Toro (Washington), Alexander Volberg (Michigan/Bonn)

Topics

The trimester program was roughly divided into two thematic halves. The first one, from January to mid-March, primarily focused on harmonic analysis, for example on problems involving singular integrals, harmonic and caloric measures, elliptic and parabolic partial differential equations, and uniform rectifiability in Euclidean and non-Euclidean spaces. The second half, accelerating towards the end of March, concentrated on geometric measure theory, for example on problems related to fractals, currents, Favard length, Kakeya type problems, and the connections between these topics and contemporary Fourier analysis. The boundary between the two halves was soft, on purpose, and many residents' stay in the program overlapped with both thematic halves.

Goals

The main informal goal of the program was to bring together experts working directly on geometric measure theory (GMT), or those aspects of harmonic analysis and partial differential equations with close ties to GMT. Specific scientific goals included making progress in the following topics (the list is meant as indicative, and far from exhaustive):

- Dirichlet and Neumann problems for elliptic and parabolic partial differential equations in domains with rough boundaries.

- Free boundary problems for harmonic and elliptic measures, in particular the connection of with rectifiability.

- Theory of parabolic uniform rectifiability, and of caloric measures in domains with rough boundaries.

- Recently initiated theory of "harmonic measures" for sets with higher-codimensional boundaries.

- The *David-Semmes conjecture* on characterising uniform rectifiability in terms of the Riesz transform.

- Unique continuation problems and quantitative propagation of smallness for harmonic functions and solutions of elliptic PDE's.

- The *Favard length problem* on characterising uniform rectifiability in terms of orthogonal projections. Connections to *Vitushkin's conjecture* on analytic capacity.

- Old problems in the fractal geometric end of GMT: the *Kakeya problem*, *Falconer's distance set problem*, and the dimension of *Furstenberg sets*.

Organization

The trimester program was roughly divided into two thematic halves. During both of them, a 1-week workshop was organised. The first workshop took place between Jan 31 – Feb 4, and was entitled *Workshop on Harmonic analysis, Singular Integrals and PDEs*.

The second workshop took place between April 4–7 and was entitled *Workshop on Geometric measure theory and Harmonic analysis*. Additionally, during the first half of the trimester, a *Winter School on Harmonic analysis and PDEs* was organised between Jan 24 – 28, featuring the following mini courses:

- Guido De Philippis (Courant Institute): *PDE constrained measures*

- Steve Hofmann (Missouri): *Quantitative rectifiability, and harmonic and caloric measure*

- Eugenia Malinnikova (Stanford): *Frequency function methods in PDE*

Results

The following list contains preprints and published articles by September 2023 which mention the trimester program in the acknowledgements.

References

[1] Tomasz Adamowicz and Katrin Fässler. Hardy spaces and quasiconformal maps in the Heisenberg group. *J. Funct. Anal.*, 284(6):Paper No. 109832, 68, 2023.

[2] Pascal Auscher and Moritz Egert. A universal variational framework for parabolic equations and systems. *arXiv e-prints*, page arXiv:2211.17000, November 2022.

[3] David Beltran, Joris Roos, and Andreas Seeger. Endpoint sparse domination for classes of multiplier transformations. *arXiv e-prints*, page arXiv:2212.12437, December 2022.

[4] Mingming Cao, Pablo Hidalgo-Palencia, and José María Martell. Carleson measure estimates, corona decompositions, and perturbation of elliptic operators without connectivity. *arXiv e-prints*, page arXiv:2202.06363, February 2022.

[5] Mingming Cao, Honghai Liu, Zengyan Si, and Kôzô Yabuta. Limited range extrapolation with quantitative bounds and applications. *arXiv e-prints*, page arXiv:2206.12570, June 2022.

[6] Alan Chang, Damian Dabrowski, Tuomas Orponen, and Michele Villa. Structure of sets with nearly maximal Favard length. *Anal. PDE (to appear)*.

[7] Alan Chang, Georgios Dosidis, and Jongchon Kim. Nikodym sets and maximal functions associated with spheres. *arXiv e-prints*, page arXiv:2210.08320, October 2022.

[8] Stefano Decio and Eugenia Malinnikova. On a Bernstein inequality for eigenfunctions. *arXiv e-prints*, page arXiv:2208.10541, August 2022.

[9] Francesco Di Plinio and Anastasios Fragkos. The weak-type Carleson theorem via wave packet estimates. *arXiv e-prints*, page arXiv:2204.08051, April 2022.

[10] Max Engelstein, Cole Jeznach, and Svitlana Mayboroda. Non-local distance functions and geometric regularity. *arXiv e-prints*, page arXiv:2208.07342, August 2022.

[11] Katrin Fässler, Andrea Pinamonti, and Pietro Wald. Kakeya maximal inequality in the Heisenberg group. *arXiv e-prints*, page arXiv:2212.01845, December 2022.

[12] Ian Fleschler, Xavier Tolsa, and Michele Villa. Faber-Krahn inequalities, the Alt-Caffarelli-Friedman formula, and Carleson's ε^2 conjecture in higher dimensions. *arXiv e-prints*, page arXiv:2306.06187, 2023.

[13] Josep M. Gallegos. Size of the zero set of solutions of elliptic PDEs near the boundary of Lipschitz domains with small Lipschitz constant. *Calc. Var. Partial Differential Equations*, 62(4):Paper No. 113, 52, 2023.

[14] Antoine Julia and Andrea Merlo. On sets with unit Hausdorff density in homogeneous groups. *Forum Math. Sigma*, 11:Paper No. e36, 16, 2023.

[15] Alejandro Molero, Mihalis Mourgoglou, Carmelo Puliatti, and Xavier Tolsa. L^2-boundedness of gradients of single layer potentials for elliptic operators with coefficients of Dini mean oscillation-type. *Arch. Ration. Mech. Anal.*, 247(3):Paper No. 38, 59, 2023.

[16] Mihalis Mourgoglou, Bruno Poggi, and Xavier Tolsa. Solvability of the Poisson-Dirichlet problem with interior data in $L^{p'}$-Carleson spaces and its applications to the L^p-regularity problem. *arXiv e-prints*, page arXiv:2207.10554, July 2022.

[17] Tuomas Orponen. Additive properties of fractal sets on the parabola. *Ann. Fenn. Math.*, 48(1):113–139, 2023.

[18] Tuomas Orponen, Pablo Shmerkin, and Hong Wang. Kaufman and Falconer estimates for radial projections and a continuum version of Beck's Theorem. *arXiv e-prints*, page arXiv:2209.00348, September 2022.

[19] Olli Tapiola and Xavier Tolsa. Connectivity conditions and boundary Poincaré inequalities. *arXiv e-prints*, pages arXiv:2205.11667, to appear in Anal. PDE, May 2022.

[20] Alexander Volberg. Pisier type inequalities for K-convex spaces. *arXiv e-prints*, page arXiv:2208.04423, August 2022.

[21] Alexander Volberg. Tail spaces estimates on Hamming cube and Bernstein–Markov inequality. *arXiv e-prints*, page arXiv:2208.00472, July 2022.

[22] Alexander Volberg and Haonan Zhang. Noncommutative Bohnenblust–Hille inequalities. *arXiv e-prints*, page arXiv:2210.14468, October 2022.

[23] Alexia Yavicoli. Thickness and a gap lemma in $\mathbb{R}^d$. *arXiv e-prints*, page arXiv:2204.08428, April 2022.