Conference

" Nonlinear PDEs, images, shapes and beyond - a conference in honor of Martin Rumpf´s 60th birthday "

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organized by
Benjamin Berkels, Alexander Effland, Benedikt Wirth

Abstracts

Mirela Ben-Chen (Technion Israel Institute of Technology)

From Thin Film Flow to Elastic Correspondence, a functional approach to surface processing

Abstract: If you take some honey and pour it on top of a cake, how will it flow? This is the question that we have set to answer almost ten years ago, and which has led us to Prof. Rumpf's inspiring work. This problem has initiated a journey during which we simulated thin films on surfaces, then ventured into elastic correspondences, eventually using elasticity to guide functional shape correspondence. I will discuss these three problems, focusing on the journey, the destination and the unexplored adventures that lie ahead.

Sergio Conti (University of Bonn)

A variational phase-field model of cohesive fracture

Abstract: I shall discuss a phase-field model of cohesive fracture, and its relation with a sharp-interface model within the framework of Gamma-convergence. A damage function which depends both on the phase field and the regularization parameter permits to reconstruct many traction-separation laws. In the limit, the energy contains an (elastic) volume energy, and an opening-dependent jump energy concentrated on the fractured surface. A key step in the construction of the recovery sequence is a finite-element approximation of SBV functions with jump set of infinite length. The talk is based on joint work with Flaviana Iurlano and Matteo Focardi.

Patrick Dondl (Albert-Ludwigs-Universität Freiburg)

Pomelo and Cactus-Mathematics

Abstract: We consider two questions related to problems arising in plant biomechanics. In the first part, we examine a discrete elastic energy system with local interactions and additional non-local random long-range interactions. Our model can be used to describe the elastic energy of standard,

homogeneous, materials that are reinforced with long-range stiff fibers: the peel of the pomelo fruit, which is extremely effective in converting a local impact into a broad and non-destructive energy dissipation, is our main inspiration from biology. We analyze the asymptotic behavior of this model as the grid size tends to zero and obtain — in an appropriate scaling regime — a standard Dirichlet energy for the local term and a fractional Laplace-type term as the long-range interactions' homogenized limit. In the second part of the presentation, We develop a finite element model to understand the abscission process of different species of cacti. In particular, the cactus species Opuntia ficus-indica and Cylindropuntia bigelovii exhibit a vastly different effective fracture toughness, while relying — as all plants do — on a very limited set of basic materials as building blocks. We thus include the available morphometric and biomechanical data of our cactus species in the variational framework of brittle fracture presented by Bourdin et. al in 2008 and study their behavior compared to fictional cacti to isolate the effect of different geometric, microstructural, and materials features on the effective fracture toughness. The results are compared to experimental testing. The main motivation of this research is to gain inspiration for novel methods aiding the separation of artificial materials systems to sort raw materials for sustainable reuse and recycling.

Martin Metscher (MTU Aero Engines AG)

Shaping the future of aviation - Technology roadmap towards emission-free flying

Abstract: Emissions-free flight is the vision that drives us. Starting with an introduction of MTU Aero Engines AG the presentation will give an insight into the development of future engine technology and MTU's technology roadmap towards emission-free flying.

Felix Otto (Max-Planck-Institut für Mathematik in den Naturwissenschaften)

Thin-film equation with thermal noise: a need for renormal ization, and a positivity-preserving discretization

Abstract: Even for small sample sizes, continuum physics – in its overdamped ver sion – remains valid but needs to account for thermal noise, guided by the fluctuation-dissipation principle. This typically leads to a forcing term by space-time white noise in the nonlinear parabolic partial differential equation. In case of thin liquid films, driven by capillarity and damped by viscosity, the noise creates a roughness that leads to a small scale divergence in the nonlinearity. This mathematical phenomenon is well-known in quantum field theories, and requires a renormalization.
Clearly, this insight has to inform numerical simulation for mesh-independance: How to coarse-grain the cut-off provided by some physics on an atomistic scale to a counter term on the level of the grid size? The thin-film equation comes with an additional challenge: The configuration space of all film heights $h \geq 0$ obviously has a boundary. How to ensure that the scheme does not introduce a spurious boundary condition?
For the latter, we appeal to the underlying gradient flow structure, which by the nature of the fluctuation-dissipation principle lifts to the case with thermal noise, and allows for a thermodynamically consistent discretization.It turns out that the discretization of the deterministic thin-film equation proposed by Grün-Rumpf can be interpreted as a mixed discretization of the gradient flow structure; its good properties in terms of positivity-preservation persist in the presence of thermal noise.
The talk relies on work with Benjamin Gess, Rishabh Gvalani, Florian Kunick, and reports on work by Gvalani with Markus Tempelmayr.

Carola-Bibiane Schönlieb (University of Cambridge)

Mathematical modelling and AI for scientific imaging: from nonlinear PDEs and Riemannian geometry to neural networks

Abstract: In this talk we will discuss mathematical problems that arise in scientific imaging, ranging from variational models and PDEs for image analysis and inverse imaging problems to discrete geodesic calculus for shape matching, as well as recent advances where such mathematical models are complemented and replaced by deep neural networks. We will particularly focus on the current interplay and tension between mathematical modelling and modern AI approaches, first and foremost deep neural networks. The talk is furnished with applications to low-dose computed tomography for cancer screening, electron microscopy for modelling protein dynamics, and fast magnetic resonance tomography for imaging fluid flow.

Peter Schröder (California Institute of Technology)

Going with the Flow

Abstract: Given a sequence of poses of a body, say, the cyclical shape change of a swimmer in a possibly moving medium such as water or air, what is the resulting motion of the body? Given the shape change, a trajectory in shape space, what governs the remaining rigid motion degrees of freedom? In this talk I will discuss an approach to this setting which treats the body as one governed by changing inertia and external forces such as those due to gravity and drag and lift due to the surrounding medium. A notable feature of our approach is that it does not simulate the fluid medium but approximates its impact based on local computations. In particular we introduce a local treatment of the added mass due to momentum transfer between body and fluid. The resulting equations generalize the standard second order equations for the motion of a rigid body. We demonstrate the efficacy of our approach by comparison with experiment.
Joint work with Yousuf Soliman, Marcel Padilla, Oliver Gross, Felix Knöppel, and Ulrich Pinkall

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Rüdiger Schultz (Universität Duiburg-Essen)

How shapes learned shaking

Abstract: Around 20 years ago, still in Duisburg, we started with Martin, and joined by Sergio Conti in 2004, to discuss the issue of uncertainty in shape optimization. In the talk we get back to the beginnings and revisit some developments mutually inspired by shape optimization and stochastic programming.

Gabriele Steidl (Technische Universität Berlin)

Wasserstein Gradient Flows and Generative Models for Posterior Sampling in Inverse Problems

Abstract: This talk is concerned with inverse problems in imaging from a Bayesian point of view, i.e. we want to sample from the posterior given noisy measurement. We tackle the problem by studying gradient flows of particles in high dimensions. More precisely, we analyze Wasserstein gradient flows of maximum mean discrepancies defined with respect to different kernels, including non-smooth ones. In high dimensions, we propose the efficient flow computation via Radon transform (slicing) and subsequent sorting. Special attention is paid to non smooth Riesz kernels which Wasserstein gradient

flows have a rich structure. Finally, we approximate our particle flows by conditional generative neural networks and apply them for conditional image generation and in inverse image restoration problem like computerized tomography.

Robert Strzodka (Universität Heidelberg)

Alternating and Multiplicative Operator Splittings

Abstract: We present an algebraic framework for operator splitting preconditioners for general sparse matrices. The framework leads to four different approaches: two with alternating splittings and two with a multiplicative ansatz. The ansatz generalizes ADI and ILU methods to multiple factors and more general factor form. The factors may be computed directly from the matrix coefficients or adaptively by incomplete sparse inversions.
The special case of tridiagonal splittings is examined in more detail. We decompose the adjacency graph of the sparse matrix into multiple (almost) disjoint linear forests and each linear forest (union of disjoint paths) leads to a tridiagonal splitting. We obtain specialized variants of the four general approaches. Parallel implementations for all steps are provided on a GPU. We demonstrate the effectiveness and efficiency of these preconditioners combined with GMRES on various matrices.

Max Wardetzky (Universität Göttingen)

Sub-Riemannian Random Walks: From Connections to Retractions

Abstract: According to a version of Donsker's theorem, geodesic random walks on Riemannian manifolds converge to the respective Brownian motion. From a computational perspective, however, evaluating geodesics can be costly. We therefore study random walks based on the concept of retractions, i.e., approximations of geodesics. In the first part of the talk, as a warm-up, we consider Riemannian manifolds, and show that these approximate walks converge to the correct Brownian motion as long as the geodesic equation is approximated (at least) up to second order. In the second part of the talk, we extend the picture to the case of sub-Riemannian manifolds. We (i) provide conditions for convergence of geodesic random walks defined with respect to so-called partial connections to the correct horizontal Brownian motion and (ii) provide examples of computationally efficient retractions in the sub-Riemannian case for simulating anisotropic Brownian motion. This is joint work with Michael Herrmann, Pit Neumann, Simon Schwarz, and Anja Sturm.

Barbara Zwicknagl (Humboldt-Universität zu Berlin)

Variational models for pattern formation in helimagnets

Abstract: We consider variational models for pattern formation in helimagnets. Starting from a frustrated ferromagnetic/anti-ferromagnetic S^1-valued spin system, we first discuss the continuum limit model at the helimagnetic/ferromagnetic transition point in the case that incompatible boundary conditions are assigned on the spin field. For the resulting singularly perturbed multiwell energy, we derive the scaling law for the minimal energy in terms of the problem parameters, and outline connections to other models for pattern-forming systems. The results indicate in particular that in certain parameter regimes the formation of various complex branching-type patterns is expected. Finally, we discuss the scaling behaviour of the minimal energy of the discrete spin model. This talk is based on joint work with Janusz Ginster and Melanie Koser (both Humboldt-Universität zu Berlin).
