HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

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organized by
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Abstracts

Dominik Beck (Charles University Prague)

Mean distance in polyhedra, an application of the Crofton Reduction Technique

Abstract: Mean distance between two randomly selected points chosen uniformly from the interior of a given convex polyhedron K was known in the exact form only for K being a cube (the Robbins constant). However, a modification of the Crofton Reduction Technique always turns the problem into finite series of solvable double integrals. This way, we managed to derive the exact mean distance for all other regular polyhedra (tetrahedron, octahedron, dodecahedron, icosahedron). As the procedure can be done for any polyhedron, the mean distance is always expressible in the exact form.

Florian Besau (Technical University Wien)

The floating body construction and its applications

Abstract: Given a d-dimensional convex body in a real vector space, the floating body construction yields a one-parameter family of convex bodies that are associated with the original body in an affine covariant way. This construction has a long history in geometry, and its origins can be traced back to the 19th century, starting with C. Dupin.
In this talk, I will briefly review the definition of floating bodies and their properties. In particular, we will explore the connection between the volume of the floating body and Blaschke's classical notion of affine surface area.
Moreover, the volume of the floating body also serves as an important geometric estimate when investigating the expected volume of random polytopes generated as the convex hull of points drawn randomly from a fixed convex body.
I will also present generalizations of the classical floating body construction, such as the weighted floating body. These generalizations allow, in particular, the transfer of many classical results into non-Euclidean spaces, such as d-dimensional spherical and hyperbolic spaces.

Jaume de Dios Pont (ETH Zürich)

Query lower bounds for log-concave sampling

Abstract: A central problem in generative machine learning is that of sampling from probability distributions with a prescribed density: Given the density of a random variable (for example, as a black-box function that one can query) generate samples from a random variable that has a distribution "similar enough" to the given one.
Significant effort has been devoted to designing more and more efficient algorithms, ranging from relatively simple algorithms, such as rejection sampling, to increasingly sofisticated such as langevin-based or diffusion based models. In this talk we will focus on the converse question: Finding universal complexity lower bounds. We will do so in the case when the log-density is a strictly concave smooth function. In this case we will be able to construct tight bounds in low dimension using a modification of Perron's sprouting construction for Kakeya sets.
Based on joint work with Sinho Chewi (IAS), Jerry Li (Microsoft Research), Chen Lu (MIT) and Shyam Narayanan (MIT).

Dylan Langharst (Sorbonne University)

Is there a triangle inequality for measures of compact, convex sets?

Abstract: If K and L are convex bodies, then K being a subset of L implies the surface area of K is less than the surface area of L. If A, B and C are also convex bodies, then the Lebesgue measure satisfies the following supermodularity inequality for their Minkowski sums: $|A+B|+|A+C| < |A|+|A+B+C|$. In this talk, we explore weighted analogues of these properties by replacing the Lebesgue measure with a nice Borel measure. Recently, G. Saracco and G. Stefani showed that if a Borel measure with density has the monotonicity property, then it must be a multiple of the Lebesgue measure. We study the case of supermodularity for any Radon measure, and show it is equivalent to a variant of the monotonicity problem. We verify that a Radon measure with the supermodularity property must be the Lebesgue measure. We then consider restricted versions of the problem.

Eli Putterman (Tel Aviv University)

On the variance and admissibility of empirical risk minimization on convex function classes

Abstract: Given noisy samples from an unknown function f belonging to a known class of functions $\mathcal{F}$, how can we estimate f so as to minimize the expected error? The idea of empirical risk minimization (ERM) is simple: just choose the $f \in \mathcal{F}$ whose vector of values at the observation points is the closest possible, among all functions in $\mathcal{F}$, to the observations. Though the ERM seems quite natural, it is known to be a minimax suboptimal estimator for some natural function classes. (The notion of minimax risk, and all other necessary background from statistics, will be fully explained in the talk.) We have recently shown, however, that under very mild assumptions, the variance of ERM is always minimax optimal, so if the ERM is suboptimal this must be because it is biased. We will give the simple proof of this fact, which boils down to a technique very familiar to convex geometers - concentration of measure for Lipschitz functions on Gauss space. If time permits, we will also explain how this result yields a simple new proof of a well-known theorem of Chatterjee that the ERM is always an admissible estimator. Based on a joint work with Gil Kur and Alexander Rakhlin.

Benedikt Rednoß (Ruhr University Bochum)

A Quantification of the Fourth Moment Theorem for Cyclotomic Generating Functions

Abstract: This talk deals with sequences of random variables X_n only taking values in $\{0, ..., n\}$. The probability generating functions of such random variables are polynomials of degree n. Under the

assumption that the roots of these polynomials are either all real or all lie on the unit circle in the complex plane, a quantitative normal approximation bound for X_n is established in a unified way. In the real-rooted case the result is classical and only involves the variances of X_n, while in the cyclotomic case the fourth cumulants or moments of X_n appear in addition. The proofs are elementary and based on the Stein-Tikhomirov method.
This is a joint work with Christoph Thäle.

Daniel Rosen (Technische Universität Dortmund)

Random Polytopes

Abstract: Random polytopes are one of the central and oldest models of stochastic geometry, their roots going back to Sylvester's famous four-point problem of the 19-th century. Their study sits at the crossroads of convex geometry, probability theory and integral geometry. This mini-course will give an introduction to the study of random polytopes, in particular the asymptotic study of their geometric and combinatorial properties. If time permits we will also study similar questions in non-Euclidean settings.

Mathias Sonnleitner (University of Passau)

Critical intersection volumes of (Schatten) p-balls

Abstract: In high dimensions concentration of measure can enforce a change of behavior at a critical threshold. One such example is the intersection volume of a unit-volume ball with respect to some norm with a rescaled ball in another norm. This phenomenon gives insight into the distribution of volume in the original convex body. We provide a brief survey of previous results and focus on the behavior at the critical threshold. There, the intersection volume is governed by the involved limiting distribution, which is Gaussian or Gumbel for classical p-balls and Tracy-Widom for certain Schatten p-balls.
Our contributions are based on joint work with C. Thäle.

Clara Stegehuis (University of Twente)

Optimization-based analysis of random graphs

Abstract: Networks have many important applications, and are often modeled by random graph models. In this mini-course, we will first focus on random graph models with different properties related to their degrees and clustering. We will first explore the general properties of these models, and will then focus on random graphs with power-law degree distributions. While typically, heavy-tailed degree distributions make it more difficult to analyze random graph properties, we will focus on an optimization-based method that is able to detect the scaling of many network properties in an intuitive, simple way. We will then show that the same method also applies to networks with an underlying geometry.

David Steigenberger (University of Münster)

Random Beta Simplices and Parallelotopes

Abstract: We study random simplices defined as convex hulls $[X_1, \dots, X_k]$ of $k \le d+1$ independent, but not necessarily identically distributed random vectors $X_1, \dots, X_k$ in $\mathbb{R}^d$ as well as random parallelotopes defined as the Minkowski sums of the random segments $[0, X_1], \dots, [0, X_k]$. For $\beta_i > -1$, each vector X_i is beta-distributed with parameter β_i, that is, it has density

$$f_{d,\beta_i}(x) = c_{d,\beta_i} \left(1 - \|x\|^2\right)^{\beta_i}, \qquad \|x\| \le 1,$$

where c_{d,β_i} is a suitable normalizing constant. Special cases of this distribution include the uniform distribution on the d-dimensional Euclidean unit ball for $\beta = 0$, the uniform distribution on the $(d-1)$-dimensional Euclidean unit sphere $\mathbb{S}^{d-1}$ as the weak limit as $\beta \downarrow -1$ and the d-dimensional Gaussian distribution as the (normalized) weak limit as $\beta \to +\infty$.
If $X_1, \dots, X_k$ are each beta-distributed with (possibly) different parameters β_i, we call their convex hull $[X_1, \dots, X_k]$ a beta simplex. In the talk, we show an explicit formula for the expected volume of such a beta simplex by using a transparent and geometric approach which connects the volumes of simplices and parallelotopes. This explicit formula can then be used to prove a distributional representation of the volume of a beta simplex, in which the aforementioned geometrical relation emerges once again. Joint work with Zakhar Kabluchko and Christoph Thäle.

Anna Strotmann (University of Osnabrück)

Poisson-Delaunay-Approximation

Abstract: The Poisson-Delaunay tesselation is a random tesselation, whose construction is based on a Poisson point process. The union of all Delaunay cells with center in some compact convex set K is called the Poisson-Delaunay approximation of K. In this talk, lower and upper bounds for the variance and a central limit theorem for the volume of the Poisson-Delaunay approximation will be presented.

Kateryna Tatarko (University of Waterloo)

Isoperimetric problem: from classical to reverse

Abstract: The well-known classical isoperimetric problem states that the Euclidean ball has the largest volume among all convex bodies in $\mathbb{R}^n$ of a fixed surface area. We will discuss the question of reversing this result for the class of convex bodies with curvature at each point of their boundary bounded below by some positive constant. In particular, we discuss the problem of minimizing the volume in this class of convex bodies and resolve it in $\mathbb{R}^3$.

Vanessa Trapp (Hamburg University of Technology)

title of abstract

Abstract: Excursion sets of Poisson shot noise processes are a prominent class of random sets. We consider a specific class of Poisson shot noise processes whose excursion sets within compact convex observation windows are almost surely polyconvex. This class contains, for example, the Boolean model. In this talk, we analyse the behaviour of geometric functionals such as the intrinsic volumes of these excursion sets for growing observation windows. In particular, we study the asymptotics of the expectation and the variance, derive a lower variance bound and show a central limit theorem.

Beatrice-Helen Vritsiou (name of University)

MM^*-estimates, regular M-ellipsoids, and distances between convex bodies

Abstract: Given a convex body K in n-dimensional Euclidean space, the width of K in a certain (unit) direction is the distance between two parallel supporting hyperplanes of K which are orthogonal to this direction. The average/mean width (over the Haar measure on the unit Euclidean sphere) is proportional to one of the intrinsic volumes of K and is an important parameter of K. Another important quantity is the product of the mean width of K and the mean width of the polar of K (here we assume that K contains the origin in its interior). Unlike the volume product (the protagonist of the celebrated Blaschke-Santaló and Bourgain-Milman inequalities, and the famous Mahler conjecture), the product of the mean widths (traditionally denoted as MM^*) is not an affine invariant of K: it can be shown, by a simple application of Jensen's inequality, together with the Blaschke-Santaló inequality, that MM^* is lower-bounded by 1, but it can get arbitrarily large. Thus, here we are interested in a 'special' position of K that minimises the quantity MM^* over all linear (or affine) images of K.
It was the culmination of several seminal results in Asymptotic Geometric Analysis that led to the very important fact that, for origin-symmetric K (that is, $K = -K$), the minimal MM^* of an image of K is upper-bounded by at most $\log(n)$. However, in the non-symmetric case the situation is still unresolved, and is considered of great interest. The quantities of mean width and of MM^* are also related to how efficiently we can cover a convex body by larger and larger dilates of the Euclidean ball (regular ellipsoids), and, in the non-symmetric case, to an approach by Rudelson that gives the best bounds we have on the maximal distance between two convex bodies in $\mathbb{R}^n$. We will give a brief introduction to some of these results, and we will discuss some subtleties and differences between the symmetric and non-symmetric cases.
