“Workshop: Asymptotics of (random) convex sets: fluctuations and large deviations”

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organized by
Kavita Ramanan, Matthias Reitzner and Christoph Thäle

Abstracts

Radek Adamczak (University of Warsaw)

Limit theorems for the volumes of small codimensional random sections of l_n^p-balls

Abstract: I will present central limit theorems for the volumes of random sections of fixed codimension of high dimensional l_n^p balls. I will also discuss some related open questions. Based on joint work with Peter Pivovarov and Paul Simanjuntak.

Pierre Calka (University of Rouen)

Close-up on random convex hulls

Abstract: In this talk, we consider the convex hull of a set of random points uniformly distributed inside a smooth convex body. When the size of the input goes to infinity, we investigate the Hausdorff distance between the random polytope and the initial smooth convex body and show that it satisfies an asymptotic expansion up to a convergence in distribution to the Gumbel law. The strategy involves the explicit calculation of the extremal index for this particular functional and the proof that it does not depend on the chosen smooth convex body. The particular case of the unit ball is treated through a reinterpretation of the problem as the estimation of a covering probability of the unit sphere by spherical caps. If time permits, we will show how the technique can also be applied in the high-dimensional context. The talk is based on joint works with Joe Yukich and with Benjamin Dadoun.

Anna Gusakova (University of Münster)

Concentration inequalities for Poisson U-statistics

Abstract: Let η be a Poisson point process on a general measurable space. A Poisson functional is a random variable $F(\eta)$, such that almost surely we have $F(\eta) = f(\eta)$ for some measurable function f on the space of counting measures. Poisson functionals have been intensively studied within last years and they play an important role in stochastic geometry since many important geometric functionals of stochastic geometry models are in fact Poisson functionals. Poisson U-statistic is an example of Poisson functional, which has particularly nice structure. In this talk a number of concentration inequalities for Poisson U-statistics under some rather mild conditions will be presented. We will discuss their optimality and consider a few applications to stochastic geometry models.

Zakhar Kabluchko (University of Münster)

Exponential f-vector profiles of high-dimensional random polytopes

Abstract: For each $d = 1, 2, \ldots$ let P_d be a d-dimensional polytope (which may be deterministic or random). Let $f_k(P_d)$ be the number of k-dimensional faces of P_d. We are interested in the asymptotic behavior of $f_k(P_d)$ when $d, k \to \infty$ such that $k/d \to \alpha$ for some $\alpha \in (0,1)$. If the limit $h(\alpha) := \lim_{d\to\infty} \frac{1}{d} \log \mathbb{E} f_k(P_d)$ exists for every $\alpha \in (0,1)$, we call the function $h(\alpha)$ the exponential profile of the sequence P_d. We will discuss the existence of an exponential profile (and some related questions and conjectures) for several natural sequences of random and deterministic polytopes. Examples include Gaussian polytopes (Vershik and Sporyshev), convex hulls of random walks (joint works with Dmitry Zaporozhets, Vladislav Vysotsky and Alexander Marynych) and Poisson zero cells.

Gil Kur (ETH Zurich)

Connections between Minimal Norm Interpolation and Local Theory of Banach Spaces

Abstract: We study the statistical performance of minimum norm interpolators in non-linear regression under additive Gaussian noise. Specifically, we focus on norms that satisfy either 2-uniformly convexity or the cotype 2 property – these include inner-product spaces, ℓ_p norms, and W_p Sobolev spaces for $1 \leq p \leq 2$. Our main result demonstrates that under 2-uniform convexity, the bias of the minimal norm solution is bounded by the Gaussian complexity of the class. We then prove an Efron-Stein type estimate for the variance of the minimal norm solution under cotype 2 or 2-uniform convexity. Our approach leverages tools from the local theory of finite dimensional Banach spaces, and to the best of our knowledge, it is the first to study non-linear models that are "far" from Hilbert spaces.

Rafal Latala (University of Warsaw)

Two-sided estimates for operator $\ell_p \to \ell_q$ norms of random matrices with iid entries

Abstract: We will present two-sided estimates for the expected value of operator norms from ℓ_p^n to ℓ_q^m (with constants independent of p, q, n and m) of random matrices with iid entries satisfying mild regularity assumptions. We will discuss main tools used in proofs and present some examples and open problems. Talk will be based on joint works with Marta Strzelecka.

Alexander Litvak (University of Alberta)

On the Rademacher projection in the non-symmetric case

Abstract: Let K be a centrally symmetric n-dimensional convex body and d_K denote the Banach-Mazur distance from K to the Euclidean ball. The Pisier bound on the norm of Rademacher projection $R : L_2(K) \to L_2(K)$ of order $\log d_K$ was a major tool in obtaining a logarithmic bound on $MM^*(K)$. Later it was shown that a properly adjusted (to the choice of center) notion of the norm of Rademacher projection (restricted to the set of mean zero functions) can be still used to bound $MM^*(K)$ in the absence of symmetry and that this norm is bounded above by $\sqrt{d_K}$ (note that in the non-symmetric case d_K can be of the order n). We prove sharpness of the bound on $\|R\|$ by showing that for every $d \leq \sqrt{n}$ there exists a convex body K such that $d_K \approx d$ and $\|R\| \geq c\sqrt{d_K}$. This is a joint work with F. Nazarov.

Sergii Myroshnychenko (University of the Fraser Valley)

A tale of Alice, Bob, and Contractions

Abstract: Suppose Alice wants to communicate with Bob using a collection of points S in space. However, the night is foggy, so Bob receives the random point $x + W$ when Alice sends x, where W is uniformly distributed on the unit ball. Does communication suffer if the points in S are brought pairwise closer together? Using methods of rearrangement and majorization, in a joint project with G. Aishwarya, I. Alam, D. Li and O. Zatarain-Vera, we affirmatively answer this Information-theoretic question in various cases. By-products of our work describe the natural behavior of intrinsic volumes of convex bodies under linear contractions.

Eliza O'Reilly (Johns Hopkins University)

Random tessellation forests: overcoming the curse of dimensionality

Abstract: Random forests are a popular class of algorithms used for regression and classification that are ensembles of randomized decision trees built from axis-aligned partitions of the feature space. However, the restriction to axis-aligned splits fails to capture dependencies between features, and random forest algorithms using oblique splits have shown improved empirical performance. To help explain the advantage of partitioning the data with oblique splits, we consider the class of random tessellations forests, generated by the stable under iteration (STIT) process in stochastic geometry, which achieve minimax optimal convergence rates for Lipschitz and C^2 functions for any fixed choice of directional distribution. In this work, we expand on the connection between the theory of stationary random tessellations and statistical learning theory to illustrate how the curse of dimensionality present in these convergence rates can be overcome in high dimensional feature space with a good choice of directional distribution for the random tessellation forest estimator.

Liran Rotem (Technion – Israel Institute of Technology)

Brunn-Minkowksi inequalities via concavity of entropy

Abstract: In recent years several conjectures were made about possible extensions of the Brunn-Minkowski inequality. These include the dimensional Brunn-Minkowski conjecture, which is only settled for rotation invariant measures. The standard approach to attacking all these extensions passes through their infinitesimal versions, and this approach requires the bodies involved to be convex.
In a recent work with Gautam Aishwarya we suggested a new approach: Instead of directly studying the measure of the sets, we study the relative entropy of random variables distributed inside the sets. In particular we study concavity properties of this relative entropy under optimal transportation. Using this technique we obtain for the first time such extended Brunn-Minkowski type inequalities for non-convex sets. New and improved log-Sobolev type inequalities also follow.

Holger Sambale (Ruhr University Bochum)

Moment Inequalities for Heavy-Tailed Distributions

Abstract: We investigate the relation between moments and tails of heavy-tailed (in particular, Pareto-type) distribution. Moreover, we derive concentration bounds for polynomial chaos like Hanson–Wright-type inequalities.

Matthias Schulte (Hamburg University of Technology)

Boolean models in hyperbolic space

Abstract: The union of the grains of a stationary Poisson process of compact convex sets in Euclidean space is called Boolean model and is a classical topic of stochastic geometry. In this talk, Boolean models in hyperbolic space are considered. They are obtained as unions of the grains of isometry invariant Poisson processes on the compact convex subsets of the hyperbolic space. Geometric functionals such as volume of the intersection of the Boolean model with a ball as observation window are studied. For increasing radius of the ball, asymptotic formulas for expectations, variances and covariances are shown and univariate and multivariate central limit theorems are derived. Compared to the the Euclidean case, some new phenomena can be observed. This talk is based on joint work with Daniel Hug and Günter Last (both Karlsruhe).

Carsten Schütt (Christian-Albrechts-University Kiel)

Affine surface area and polytopal approximation

Abstract: We give an overview of results on polytopal approximation of convex bodies involving the affine surface area. In almost all cases one assumes that the boundary of the convex body that is to be approximated is at least twice continuously differentiable, while it should be sufficient to assume that the boundary is twice differentiable in the generalized sense. We discuss this.

Maud Szusterman (Tel Aviv University)

extremizers in Fenchel and Bezout-type inequalities

Abstract: Soprunov, Zvavitch and Saroglou studied affine invariant ratios involving mixed volumes, and characterized the simplex as the unique minimizer of one of them: a similar characterization is conjectured for the smaller invariant. For this latter ratio, maximizers correspond to equality cases in Fenchel inequality. For the former ratio, maximizers include the cube but not the cross-polytope, and finding all the maximizers remains open.

Tomasz Tkocz (Carnegie Mellon University)

Resilience of cube slicing in l_p

Abstract: I shall present an extension of Ball's cube slicing result to l_p spaces for large p. Based on joint work with Eskenazis and Nayar.

Jacopo Ulivelli (TU Vienna)

Kubota-type formulas and supports of mixed measures

Abstract: Motivated by a problem for mixed Monge–Ampere measures of convex functions, we investigate the support of mixed surface area measures involving lower-dimensional disks. In the process, we establish a Kubota-type formula for mixed area measures and address a special case of a conjecture of Schneider. Joint work with Daniel Hug and Fabian Mussnig.

Beatrice Vritsiou (University of Alberta)

On the illumination conjecture for convex bodies with many symmetries

Abstract: We will discuss the Hadwiger-Boltyanski illumination Conjecture, and show how to verify it (along with its equality cases) for 1-symmetric convex bodies of all dimensions and some cases of 1-unconditional convex bodies as well. This is joint work with Wen Rui Sun.

Vladislav Vysotskiy (University of Sussex)

The isoperimetric problem for convex hulls and the large deviations rate functions of random walks

Abstract: We prove the large deviations principle for the area of convex hulls of a planar random walk, and then study the asymptotic shape of the most likely trajectories resulting in such large deviations. If the increments of the walk have a finite Laplace transform, then such a scaled limit trajectory h solves an anisotropic inhomogeneous isoperimetric problem for the convex hull of h, where the usual length is replaced by the large deviations rate functional $\int_0^1 I(h'(t))dt$ with I being the rate function of the increments. Assuming that the distribution of increments is not contained in a half-plane, we show that the optimal trajectories are smooth, convex, and satisfy the Euler–Lagrange equation, which we are able to solve explicitly for every I. Our solution resembles that of the isoperimetric problem in the Minkowski plane found by Busemann (1947).
