Report on the Dual Trimester Program

Boolean Analysis in Computer Science

September 9 - December 18, 2024

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Topics

This Dual Trimester Program focused on the connection between mathematical methods and the foundational challenges of theoretical computer science. The program brought together researchers in computer science and various areas of mathematics, including analysis, probability, and combinatorics. The core topics of the trimester program were:

- Learning theory;
- Complexity of classical and quantum algorithms;
- Vector valued functions on the hypercube;
- Complex hypercontractivity;
- Polynomial inequalities on the hypercube;
- Discrete approximation theory on the hypercube;
- Information-theoretic inequalities;
- Lattice problems.

Goals

The overarching goals of the Dual Trimester Program were to bridge theoretical computer science and mathematics, with the objective not only to foster interdisciplinary collaboration but also to advance both fields. In particular, the program aimed to utilize mathematical tools from analysis, probability, and combinatorics to solve challenging open problems recently raised in

computer science. Conversely, it sought to learn and further extend methods developed in computer science to develop new directions in mathematics. The specific aim was to explore the growing connections between harmonic analysis and theoretical computer science, especially as quantum computing and machine learning expand to spaces beyond the hypercube. Key scientific goals included learning about the recent developments on the core topics by exploring them in depth across various scientific events, and making research progress on the open problems. In particular, a goal of the introductory school was to provide an accessible introduction to the key areas such as the techniques necessary for PAC (probably approximately correct) learning, including extensions of Kahn-Kalai-Linial type theorems and recent achievements based on the Bohnenblust-Hille inequality and its quantum variant.

Organization

The program structure was organized around an introductory school, three research workshops, and a regular seminar series.

- **School: PAC (probably approximately correct) learning and Boolean Harmonic Analysis** (September 16 - 20, 2024)

  The school aimed at exploring interactions between learning theory and harmonic analysis, driven by the mathematics of quantum computing. The motivating question was how to learn complicated functions or large matrices using a small number of random (quantum) queries, assuming specific Fourier-type restrictions on the function.

  The techniques covered in the school include classical maximal influence approach to PAC learning and recent developments using Bohnenblust-Hille inequality and its quantum counterpart. The following lecture series were given:

  Inverse results for isoperimetric inequalities (Noam Lifshitz)

  Signal recovery, restriction theory, and applications (Alex Iosevich)

  Structure theorems in Boolean Harmonic Analysis (Yuval Filmus)

  Functional inequalities in Metric Geometry (Alexandros Eskenazis)

  Quantum and classical low degree learning via a dimension-free Remez inequality (Alexander Volberg).

These lectures provided participants with the essential theoretical framework required to study open problems in learning theory and complexity. In addition to the lecture series, 5 talks were given by the junior researchers Cristian González-Riquelme, Joseph Slote, Lars Becker, Francisco Escudero Gutiérrez, Miriam Gordin.

- **Workshop: Analysis and Geometry on Discrete Spaces** (October 7 - 11, 2024).

  This workshop focused on analytic questions of a discrete nature with a geometric component, including Boolean analysis, vector-valued harmonic analysis, metric embeddings, geometry of graphs and groups, and aspects of discrete probability and theoretical computer science.

  In this workshop, 19 lectures were given. The speakers were Florent Baudier, David Beltran, Pandelis Dodos, Polona Durcik, Michael Dymond, Yuval Filmus, Li Gao, Tuomas Hytönen, Guy Kindler, Vjekoslav Kovač, Jose Ramon Madrid Padilla, Stefanie Petermichl, Joris Roos, Justin Salez, Rocco Servedio, Lenka Slavíková, Błażej Wróbel, Quanhua Xu, Haonan Zhang.

- **Workshop: Analysis in TCS: testing, learning, and complexity** (November 4 - 8, 2024)

  This workshop focused on deepening the connections between harmonic analysis on the hypercube and theoretical computer science. It explored how theoretical computer science has inspired challenging new questions in analysis, particularly as quantum computing and machine learning expand to spaces beyond the hypercube. The schedule allocated time for participants to engage in research by discussing open questions and collaborating on problems.

  In this workshop, 13 lectures were given. The speakers were Srinivasan Arunachalam, Francisco Escudero Gutierrez, Tom Gur (online), Hamed Hatami, Pooya Hatami, Ohad Klein, Dmitry Krachun, Avichai Marmor (online), Dan Mikulincer (online), Shivam Nadimpalli, Joris Roos, Ohad Sheinfeld, Kewen Wu.

- **Workshop: Information theory, Boolean functions, and lattice problems** (November 18 - 22, 2024)

This event was concerned with the recent developments and innovative techniques in the areas of Boolean analysis, information theory, and lattices. This workshop also featured a discussion session about open problems and a dedicated time for collaborative research.

In this workshop, 15 lectures were given. The speakers were Gautam Aishwarya, Dario Cordero-Erasquin, Thomas Courtade, Felipe Gonçalves, James Melbourne, Chandra Nair, Piotr Nayar, Emma Pollard, Igal Sason, Lisa Sauermann, Joseph Slote, Noah Stephens-Davidowitz, Sergey Tikhonov, Tomasz Tkocz, Bruno Volzone.

- A **Trimester Seminar Series** was held throughout the program, with talks occurring multiple times per week, generally on Tuesdays and Thursdays from 2:45 to 4:00 pm.

  The series featured 19 talks and included lectures such as:

  On the asymptotics of the optimal constants in the Khinchine-Kahane inequality by Krzysztof Oleszkiewicz (September 12, 2024)

  Testing monotonicity from quantum data by Joseph Slote (October 29, 2024)

  Counting Lattice Points in Ellipsoids and the Central Limit Theorem for Quadratic Forms by Friedrich Götze (October 31, 2024)

  Sharp Strichartz Estimates via Hermite Polynomials and Hypercontractivity by Felipe Gonçalves (November 14, 2024)

  Jackson's inequality on the hypercube by Xinyuan Xie (December 5, 2024)

  The other speakers in the seminar were Devraj Duggal, Rafał Latała, Miquel Saucedo, Guy Kindler, Marco Fraccaroli, Diogo Oliveira e Silva, Jaume de Dios Pont, Alexander Borichev, Gautam Aishwarya, Egor Kosov, Gennady Uraltsev, Lars Becker, Pierre Bizeul, Benjamin Jaye.

Results

The program fostered extensive knowledge exchange through its structured events, which collectively involved 57 lecturers across the School and three dedicated Workshops. Research presented in the seminars and workshops demonstrated recent progress across the core topics. The School centered on

learning and Boolean harmonic analysis featured 5 lecturers providing formal introductions to key areas and 5 additional talks from junior researchers. The three workshops contributed an additional 47 lecturers who presented recent research. Furthermore, the Trimester Seminar Series supplemented these events with 19 individual research talks scheduled throughout the program.

The program included a substantial number of long-term participants, some staying for the entire duration (9.9.2024 - 18.12.2024), such as Dmitry Grigoryev, Guy Kindler, Diogo Oliveira e Silva, Miquel Saucedo, Alexander Volberg, and Xinyuan Xie. The format of the program, outside of the formal scheduled events like the School and Workshops, was crucial for ensuring ample time for continuous research and individual and group work on the challenging open problems that motivated the program.

The program has been successful in fostering exchange of methods that address key challenges in theoretical computer science, harmonic analysis, probability theory, and combinatorics. The program yielded various collaborations and produced several preprints and research findings. The program website lists the following preprints associated with the program:

Learning junta distributions and quantum junta states, and QAC0 circuits by F. Escudero-Gutiérrez.

Testing and learning structured quantum Hamiltonians by S. Arunachalam, A. Dutt, and F. Escudero-Gutiérrez.

Quantified Cramér-Wold Continuity Theorem for the Kantorovich Transport Distance by S.G. Bobkov and F. Götze.

Fisher-type information involving higher order derivatives by S.G. Bobkov.

On trilinear singular Brascamp-Lieb integrals by L. Becker, P. Durcik, and F. Yu-Hsiang Lin.

On the optimal Lp-L4 Khintchine inequality by A. Barański, D. Murawski, P. Nayar, and K. Oleszkiewicz.

Concavity principles for weighted marginals by D. Cordero-Erausquin and A. Eskenazis.

Beyond the listed preprints, participants initiated investigations into areas such as metric Ramsey theory, local Banach space theory, quantum learning problems, sharp restriction theory, isoperimetric inequalities on the hypercube, and new approaches to structure theorems on the symmetric group.