HAUSDORFF RESEARCH INSTITUTE
FOR MATHEMATICS

UNIVERSITÄT BONN

Follow-Up-Workshop to TP

"Periods in Physics, Number Theory and Algebraic Geometry"

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organized by
Spencer Bloch (Chicago), Herbert Gangl (Durham), Vasily Golyshev (Trieste), Fernando Rodriguez Villegas (Trieste), Don Zagier (Bonn)

Abstracts

Hohto Bekki (MPIM Bonn)

The Shintani-Barnes cocycle and special values of zeta functions of algebraic number fields

Abstract: The classical Hecke integral formula expresses the zeta functions of number fields (of degree n) as an integral of the Eisenstein series over a certain torus of $\mathrm{GL}(n)$. In the case where the number field is totally real or totally imaginary, it has been known that such an integral formula can be interpreted cohomologically using the Eisenstein cocycles or the Shintani cocycles, and one of my aims during the Trimester Program in 2018 was to obtain some ideas for extending such cohomological interpretation to general number fields. In this talk, I would like to report on the construction of a new Eisenstein cocycle called the Shintani-Barnes cocycle with which we can give a cohomological interpretation of Hecke's integral formula for general number fields.

Johannes Broedel (Humboldt-Universität zu Berlin)

Numerical approach to polylogarithms on higher-genus Riemann surfaces

Abstract: Polylogarithms on higher-genus Riemann surfaces have been constructed in various languages recently. From a particle physicists perspective, this structural knowledge has to be accompanied by the ability to numerically evaluate those functions. I will discuss how to facilitate the Schottky parametrization in order to numerically evalulate polylogarithms on a genus-two surface, thereby making use of an averaging procedure over genus-one differential forms.

Steven Charlton (MPIM Bonn)

Depth reductions of multiple polylogarithms

Abstract: I will review the setup surrounding Goncharov's Depth Conjecture, and types of the multiple polylogarithm depth reductions it predicts. I will then discuss some recent results (of subsets of Gangl, Matveiakin, Radchenko, Rudenko, and myself) proving identities and reductions in this direction.

Rob de Jeu (Vrije Universiteit Amsterdam)

K_2 of elliptic curves over non-Abelian cubic and quartic

Abstract: After a review of some earlier results on mostly K_2 of curves, we give constructions of families of elliptic curves over certain cubic or quartic fields with three, respectively four,integral elements in the kernel of the tame symbol on the curves. The fields are in general non-Abelian, and the elements inearly independent. For their integrality, we discuss a new criterion that does not ignore any torsion. We also verify Beilinsons conjecture numerically for some of the curves. This is joint work with François Brunault, Liu Hang, and Fernando Rodriguez Villegas.

Robin de Jong (Leiden University)

Heights on curves and limits of Hodge structures

Abstract: We exhibit a precise connection between Néron–Tate heights on smooth curves over number fields and biextension heights of limit mixed Hodge structures associated to smoothing deformations of singular quotient curves. Our approach suggests a new way to compute Beilinson–Bloch heights in higher dimensions. This is joint work with Spencer Bloch and Emre Sertöz.

Hidekazu Furusho (Nagoya University)

p-adic hypergeometric function and p-adic multiple polylogarithms

Abstract: I will introduce a (maybe new) p-adic analogue of Gauss hypergeometric function, by using Ohno-Zagier formula for p-adic multiple polylogarithms. I will explain my method of its residue-wise analytic prolongation and then show that a p-adic analogue of Gauss hypergeometric theorem holds.

Luis Garcia (University College London)

Elliptic units for complex cubic fields

Abstract: The elliptic Gamma function –an elliptic version of the ordinary Gamma function– is a meromorphic special function in three variables that mathematical physicists have shown to satisfy modular functional equations under SL(3, Z). In this talk I will present evidence (numerical and theoretical) that products of values of this function are often algebraic numbers that satisfy explicit reciprocity laws and are related to derivatives of Hecke L-functions of cubic fields at s = 0. We will discuss the relation to Stark's conjectures and will see that this function conjecturally allows to extend the theory of complex multiplication to complex cubic fields as envisioned by Hilbert's 12th problem. The talk will be based on arxiv:2311.04110 and is joint work with Nicolas Bergeron and Pierre Charollois.

Aleksander Horawa (University of Oxford)

Beilinson's conjecture for the Dwork family

Abstract: (Joint work with Lambert A'Campo.) The Dwork family Y_t given by the equation $x_0^5 + \cdots + x_4^5 = 5tx_0 \dots x_4$ in $\mathbb{P}^4$ has rich arithmetic structure. For example, Harris–Shepherd-Barron–Taylor

showed that certain submotives M_t of $H^3(Y_t)$ are *potentially modular*: their Galois representations are associated with automorphic forms for $\mathrm{GSp}_{4,K}$ for a finite extension $K/\mathbb{Q}$ and it is expected that we could take $K = \mathbb{Q}$. In particular, the motivic L-function $L(M_t, s)$ is related to an automorphic L-functions and hence has good analytic properties. In ongoing work, we study Beilinson's conjecture for the L-value $L''(M_t, 0)$: the higher Chow group $\mathrm{CH}^4(M_t, 4)$ should have rank two and the regulator of its elements should be related to $L''(M_t, 0)$. We will discuss progress towards constructing the two desired classes in $\mathrm{CH}^4(M_t, 4)$ and computing their regulators.

Axel Kleinschmidt (MPI for Gravitational Physics, Potsdam)

Iterated integrals of modular forms in string theory

Abstract: The low-energy expansion of closed string scattering amplitudes at one-loop order leads to non-holomorphic modular forms of $SL(2, \mathbb{Z})$. These can be expressed through iterated (Eichler type) integrals of holomorphic modular forms, their complex conjugates and interesting periods that were called multiple modular values by F. Brown. I will present the physics perspective on these ideas based on work with Daniele Dorigoni, Mehregan Doroudiani, Joshua Drewitt, Martijn Hidding, Oliver Schlotterer, Leila Schneps and Bram Verbeek.

Albrecht Klemm (Universität Bonn)

Fishnet Theories, Yangian Symmetries and Calabi-Yau volumes

Abstract: 2d Fishnet Theories exhibit Yangian integrable symmetries which constrain their amplitudes. We show that the latter are calibrated volumes of associated Calabi-Yau varieties. These volumes are given by the Griffiths non-holomorphic period bilinears. The corresponding periods are solutions of the flat Gauss-Manin connection or equivalent the Picard-Fuchs differential ideal in a special integral monodromy representation. The Picard-Fuchs differential ideal is equivalent to the semi direct product of the Yangian over the conformal algebra and the automorphism group of the underlying graph. We discuss graphs associated to hexagonal, quadratic and triangular tilings of the plane, the corresponding Calabi-Yau varieties and discuss how their geometries are related under additional symmetries of the theory. The formalism leads also to an efficient analytic evaluation of multi parameter fishnet integrals.

Hossein Movasati (IMPA)

Detecting Gauss-Manin and Calabi-Yau differential equations

Abstract: In this talk I will review few conjectures which aim to detect which linear differential equations come from Gauss-Manin connections, that is, they are satisfied by periods of families of algebraic varieties. This includes conjectures due to Katz-Grothendieck, André and Bombieri-Dwork. I will discuss another finer criterion to detect differential equations coming from families of hypergeometric Calabi-Yau varieties. Finally, I will explain a classification list in the case of Heun and Painlevé VI equations.

Bartosz Naskręcki (Adam Mickiewicz University in Poznań)

Moments of families of elliptic curves

Abstract: In this talk we will discuss some new developments in higher moment sums of 1-parametric families of elliptic curves. These sums have connections to modular forms and algebraic curves. I will sketch a result about the second moment of cubic curves leading to a connection with intermediate Jacobians in threefolds. Next we will discuss proofs of modularity of certain rigid Calabi-Yau threefolds which uses directly higher moments, universal families of elliptic curves and Deligne's results, avoiding completely the standard approach via Faltings-Serre method.

Wieslawa Niziol (CNRS/Sorbonne Université)

Hidden structures on de Rham cohomology of p-adic analytic varieties

Abstract: I will survey what is known about extra structures (Hodge filtration, Frobenius, monodromy) appearing on de Rham cohomology of analytic varieties over local fields of mixed characteristic.

Danylo Radchenko (Laboratoire Paul Painlevé)

Multiple polylogarithms and the Steinberg module

Abstract: I will talk about a surprising connection between the Steinberg module of rationals and a certain space of multiple polylogarithms on a torus. I will then describe some applications of this relation to the Goncharov program. In particular, I will show how this relation can be used to prove that all multiple polylogarithms of given depth and weight are generated by a single function. The talk is based on a joint work in progress with Steven Charlton and Daniil Rudenko.

Nobuo - Sato (National Taiwan University)

Iterated Beta Integrals

Abstract: I will explain my recent work on iterated beta integrals with Minoru Hirose at Nagoya University. Iterated beta integrals are a type of iterated integrals on branched universal abelian coverings of the projective line, which commonly generalize hyperlogarithms and beta integrals. In addition to the properties that generalize those of hyperlogarithms and beta integrals, iterated beta integrals enjoy a new property that we call translation invariance. The translation invariance yields equalities between iterated integrals on translation-equivalent coverings of the projective line, and especially for genus-zero pair of translation-equivalent coverings, it yields non-trivial equalities between special values of hyperlogarithms. Such equalities include the famous 2-3-2 formula for multiple zeta values proved by Zagier, its analog for multiple t-values, the 2-1 formula for multiple zeta star values, and various other formulas of a similar kind. By classifying all genus-zero pairs, some new interesting cases popped up, one of which resolves a conjecture by Charlton concerning the area of a family of Lawson surfaces.

Oliver - Schlotterer (Uppsala University)

Constructing polylogarithms on higher-genus Riemann surfaces

Abstract: In this talk, the Brown-Levin construction of elliptic polylogarithms is generalized to Riemann surfaces of arbitrary genus. Homotopy-invariant iterated integrals on a higher-genus surface are generated from a flat connection with simple poles in two marked points. The integration kernels of the flat connection consist of modular tensors, built from convolutions of the Arakelov Green function and its derivatives with holomorphic Abelian differentials. At genus one, these convolutions reproduce the Kronecker-Eisenstein kernels of elliptics polylogarithms and modular graph forms. I will report on work in progress on relations to the higher-genus polylogarithms of Enriquez-Zerbini as well as tensorial generalizations of Fay identities among our integration kernels.

Emre Can - Sertöz (Leiden University)

Computing linear relations between 1-periods

Abstract: I will sketch a modestly practical algorithm to compute all linear relations with algebraic coefficients between any given finite set of 1-periods. This is based on the "qualitative description" of these relations by Huber and Wüstholz. We combine this result with the recent work on computing the endomorphism ring of abelian varieties by Bruin, Costa, Mascot, Lombardo, Sijsling, Voight, Zotine, and others. This is a work in progress with Jöel Ouaknine and James Worrell.

Masha Vlasenko (Institute of Mathematics of the Polish Academy of Sciences)

Frobenius structure and p-adic zeta function

Abstract: For an ordinary linear differential equation a p-adic Frobenius structure is an equivalence between the local system of its solutions and its pullback under the map $t \mapsto t^p$ which is realized over the field of p-adic analytic elements. Its existence is a strong property, we only expect it for differential equations arising from the Gauss–Manin connection in algebraic geometry. In a vicinity of a singular point such a structure can be described by a bunch of p-adic constants. We will show examples of families of hypersurfaces for which these constants turn out to be p-adic zeta values. This is joint work with Frits Beukers.

Jan Vonk (Leiden University)

p-adic height pairings of geodesics

Abstract: I will discuss a certain p-adic height pairing of real quadratic geodesics on modular curves. The motivation for studying this pairing comes from its relation to real quadratic (RM) singular moduli. I will discuss how the interpretation of this height pairing as a triple product period sheds light on the conjectures that were made when RM singular moduli were defined. This is joint work with Henri Darmon.

Federico Zerbini (Uned University Madrid)

Conical sums

Abstract: Conical sums are periods defined by certain infinite sums over lattice points contained in cones of $\mathbb{R}^n$. Special cases include multiple zeta values, as well as Matsumoto-Witten zeta values associated with semisimple Lie algebras. They have also appeared in the computation of string theory amplitudes. The $\mathbb{Q}^{\mathrm{ab}}$-algebra generated by conical sums was proven by Terasoma to coincide with the $\mathbb{Q}^{\mathrm{ab}}$-algebra of cyclotomic multiple zeta values; all relations in this algebra are conjectured to follow from decompositions of cones. Open questions on Matsumoto-Witten zeta values would be answered by proving a general conjecture of Dupont about the motivic nature of conical sums. I will introduce conical sums and their algebra, and report on the current state of the art on Dupont's conjecture.

Wadim Zudilin (Radboud University Nijmegen)

Modular regulators and multiple modular values

Abstract: I will report on recent work with François Brunault, in which we explicitly computed the Goncharov regulator integral for K_4 classes on modular curves in terms of multiple modular values and reduced the latter to single L-values of modular forms.
