"School on Analysis and geometry on groups and spaces"

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organized by
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Abstracts

David Bate (University of Warwick)

An introduction to rectifiability in metric spaces

Abstract: Geometric measure theory studies geometric properties of non-smooth sets. The key concept is that of an n-rectifiable set, which can be parametrised by countably many Lipschitz images of n-dimensional Euclidean space. Characterisations of rectifiable subsets of Euclidean space have important consequences in the theory of partial differential equations, harmonic analysis and fractal geometry.
The recent interest in analysis in non-Euclidean metric spaces naturally leads to questions regarding geometric measure theory in this setting. This talk will give an overview of work in this direction. After introducing the necessary background, we will present recent characterisations of rectifiable subsets of an arbitrary metric space in terms of non-linear projections and tangent spaces.

Panos Papasoglu (University of Oxford)

An Introduction to Systolic Geometry

Abstract: I will give a quick introduction to systolic geometry. The systole of a Riemannian manifold is the length of the shortest non-contractible loop.
One seeks an upper bound in terms of the volume of the manifold. The first such result was obtained by Loewner in 1949 in the case of the torus, and Berger in the 60's asked for a generalisation for aspherical manifolds.
I plan to explain two results of Gromov in some detail:

1. The proof of the upper bound for the systole of high genus surfaces. I will give Kodani's proof.

2. The recent proof by Nabutovsky for the upper bound of the systole of an aspherical manifold – which also significantly improves the constant in the original proof.

Regina Rotman (University of Toronto)

Quantitative Topology and Geometric Inequalities

Abstract: I will discuss the quantitative versions of some existence theorem in Geometric Calculus of Variations, originally proven using the methods of Algebraic Topology. Examples of such theorems are the following: a theorem of A. Fet and L. Lyusternik, stating that on any closed Riemannian manifold there exists a periodic geodesic; a theorem of J. P. Serre, stating that for any pair of points on a closed Riemannian manifold there exist infinitely many geodesics, connecting them; a recent result of X. Zhou about the existence of a geodesic chord in a complete manifold M orthogonal to a closed submanifold N, modulo the existence of a relevant sweepout.
In particular, the quantitative versions of these theorems result in many geometric inequalities, relating the "size" of the minimal object to the geometric properties of the ambient space, such as volume, diameter, curvature.

Daniele Semola (University of Vienna)

Ricci curvature and fundamental group

Abstract: It has been known since the Forties that fundamental groups of complete Riemannian manifolds with nonnegative Ricci curvature satisfy certain restrictions. The goal of this mini-course will be to review some of these restrictions and to discuss their optimality. The topics will include:

- Structure of fundamental groups of manifolds with nonnegative Ricci curvature (polynomial growth, virtual nilpotency, local uniform finite generation, . . .);
- Discussion about some motivating examples;
- Methods to prove finite generation of fundamental groups (Gromov's short generators, Sormani's linear growth theorem, equivariant analysis of blow-downs, . . .);
- Spaces/manifolds with infinitely generated fundamental groups;
- Gluing constructions preserving nonnegative Ricci curvature;
- Construction of counterexamples to the Milnor conjecture.
