
Conference
“Definability and Computability”

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organized by
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Abstracts

Matthias Aschenbrenner (University of Vienna)

Second-order linear differential equations over Hardy fields

Abstract: Hardy fields are one-dimensional relatives of o-minimal structures. We recently proved a theorem which permits the transfer of first-order statements between Hardy fields and related domains for “tame” asymptotic analysis. I will focus on the special role played by second-order linear differential equations in this story, as well as applications of our main result to such equations.
(Joint work with L. van den Dries and J. van der Hoeven.)

Peter Cholak (University of Notre Dame)

The Henson graphs: colorings and codings

Abstract: By recent work of Dobrinen and Balko, Chodounský, Dobrinen, Hubička, Konečný, Vena, L. and Zucker, we know that every finite G in the Henson graph $\mathbb{H}_{\kappa+\aleph}$ (the universal ultrahomogeneous n -clique free graph) has exact finite big Ramsey degree $k(G, n)$. That is, there is a positive integer $k(G, n)$ such that for all finite colorings C of the copies of G in \mathbb{H}_{n+1} , there is $\tilde{\mathbb{H}}$, a substructure of \mathbb{H}_{n+1} and isomorphic to \mathbb{H}_{n+1} , such that in $\tilde{\mathbb{H}}$ at most $k(G, n)$ colors are used on the induced coloring from \mathbb{H}_{n+1} on the copies of G in $\tilde{\mathbb{H}}$. Moreover, for exactness, for some coloring and all corresponding $\tilde{\mathbb{H}}$, all $k(G, n)$ colors are needed. One result by Cholak, Dobrinen, and McCoy, we will discuss here is that if $|G| \geq 2$, then there is a finite computable coloring C such that, for all such $\tilde{\mathbb{H}}$, we have that $\tilde{\mathbb{H}}$ computes $\emptyset^{(|G|-1)}$ (and hence the halting set). We will also provide a template so that this result should be duplicatable in other structures which have finite big Ramsey degrees. We will also briefly mention another recent result by Cholak, Dobrinen, and Townser, that $\tilde{\mathbb{H}}$ can always be found arithmetically from the coloring. Also, initially, we will explain why these results might be interesting to someone not familiar with Structural Ramsey Theory.

Barbara Csima (University of Waterloo)

Measuring complexity of structures via their Scott Sentences

Abstract: Scott's Isomorphism Theorem shows that each countable structure can be uniquely defined, up to isomorphism, by a sentence of infinitary logic, now known as the Scott Sentence of the structure. The complexity of a structure's Scott Sentence can then be viewed as a measure of complexity of the structure. In this talk, we will discuss the relationship of Scott complexity with other measures of complexity, as well as discuss the Scott Complexity of certain structures.

David Gonzalez (University of Notre Dame)

Scott Filtrations and Vaught Ordinals

Abstract: Scott analysis is a branch of computable structure theory and descriptive set theory that aims to calibrate the complexity of the isomorphism relation. Two of the most fundamental notions studied in Scott analysis are Scott rank and the ordinal indexed back-and-forth relations. The Scott rank of a countable structure tells you how difficult it is to understand the isomorphism type of that structure. Meanwhile, the ordinal indexed back-and-forth relations measure how difficult it is to tell different structures apart from each other. We introduce the concept of a Scott filtration, which connects these two fundamental concepts in Scott analysis. Given a structure M , a Scott filtration uses simpler structures (i.e., structures of smaller Scott rank) to concretely understand the back-and-forth classes of M . We will discuss the existence of Scott filtrations up to the Vaught ordinal along with some other related theorems. This is joint work with Dino Rossegger and Dan Turetsky. The speaker has happily able to attend the first four weeks of the HIM DDC trimester, and thanks the institute for the support provided that helped make this project possible.

Alexi Block Gorman (University of Amsterdam)

Decidability via Symbolic Dynamics

Abstract: Shift spaces of sequences (including those indexed by the natural numbers, the integers, binary trees, etc.) are the central object of study in symbolic dynamics, and those which correspond to finite automata have useful and compelling properties. One property of recent interest is the Hom Shift, a shift which corresponds to an undirected graph in a canonical way. We will discuss the decidability of when a one-sided edge shift is conjugate to a one-sided Hom-shift, and the decidability of whether a tree edge shift is conjugate to a Hom tree shift. By interpreting these shifts in the natural way vis-à-vis k -automatic structures, we obtain analogous decidability results for classifying certain kinds of k -automatic expansions.

Ellen Hammatt (TU Wien)

Exploring structural aspects of punctual degrees

Abstract: The study of punctual structures considers what happens to the computation of presentations of structures and their isomorphisms when we forbid the use of unbounded search, i.e., the restriction to primitive recursion. We will introduce punctual degrees which are degree structures formed by considering the collection of all punctual presentations of a fixed structure under the primitive recursive isomorphisms. Notice that since the inverse of a primitive recursive function is not necessarily primitive recursive, primitive recursive isomorphisms is in fact a reduction rather than an equivalence relation. This gives us a new phenomenon that is not present in the computable structure theory world. We will discuss the current knowledge of the structural aspects of punctual degrees including density and lattice embeddings, as well as punctual dimension in relation to the punctual degrees.

Martin Hils (University of Münster)

Beautiful pairs of algebraically closed valued fields, and non-standard Frobenius

Abstract: Generalizing work of Poizat in the stable case, motivated by the definability status of important spaces of definable types in the theory ACVF of algebraically closed valued fields, Cubides Kovacsics, Ye and I introduced and studied beautiful pairs for unstable theories, as a more semantic way to approach such definability questions. As one of the main applications, we could establish the strict pro-definability of all definable types and of all bounded definable types, living on an algebraic variety V defined over the valued field - these are definable analogues of the Zariski-Riemann space and the Huber space associated to V .

In the talk, I will explain the main ideas behind this approach, which brings to bear an Ax-Kochen-Ershov type reduction. Moreover, I will report on some ongoing project, joint with Hrushovski, Ye and Zou, on versions of the results for ACVF where a non-standard Frobenius automorphism is added to the structure. We obtain in particular suprisingly simple axioms for the resulting pairs, as these turn out to be beautiful as well.

Meng-Che Ho (New College of Florida)

Isomorphism problems as equivalence relations

Abstract: A major theme in mathematics is the classification problem: given a class of objects and a notion of equivalence, classify the objects up to the equivalence. In algebra, this often takes the form of isomorphism problems. There are multiple ways to formalize isomorphism problems using computability theory, many of which are closely related to the definability of the structures. For instance, the index set of a structure is related to its infinitary definability, namely, Scott analysis; and a result of Harrison-Trainor, Melnikov, Miller, and Montalban relates computable functors between (the isomorphism classes) of structures and effective (infinitary) interpretability. In this talk, we consider a variant of the isomorphism problem that originates in algebra, namely, the isomorphism problem of (algebraic) presentations. More precisely, given an algebraic variety (in the sense of universal algebra), we consider the isomorphism problem on the finitely generated c.e. presentations in the variety. We then compare the complexities of these isomorphism problems under computable reducibility, an effectivization of Borel reducibility. We first focus on the varieties of commutative monoids, commutative semigroups, abelian groups, and unary structures. We then show some general results connecting the algebraic properties of varieties and the complexities of their isomorphism problem.

Salma Kuhlmann (University of Konstanz)

Recursively saturated real closed fields and models of Peano arithmetic

Abstract: Integer parts of real closed fields coincide exactly with models of the fragment Open Induction of Peano arithmetic (PA). In the first part of the talk, we introduce IPA-real closed fields, that is, real closed fields which admit an integer part (IP) which is a model of Peano Arithmetic (PA), and relate them to models of real exponentiation. In particular, we give an explicit description of their value groups, with respect to the natural valuation. IPA real closed fields are recursively saturated, and the converse holds in the countable case. In the second part of the talk, we provide a valuation theoretic characterization of recursively saturated real closed fields in terms of their value groups, residue fields, and the pseudo convergence of distinguished pseudo-Cauchy sequences. In the final part of the talk, we conclude by drawing the bridge between part I and part II to obtain an explicit description of countable IPA-real closed fields. Joint work with M. Carl, P. D'Aquino and K. Lange

Chris Laskowski (University of Maryland)

Classifying first order theories by Borel reducibility: Status Report

Abstract: We highlight what is known and what remains to be known about the Borel complexity of classes of countable models of first-order theories. The dividing lines for classifying countable models of a theory are significantly different from those for analyzing uncountable models, hence new tools need to be developed. We discuss the methods of potential canonical Scott sentences, groundedness, flat structures, and admitting nested sequences. Many examples, some rather unexpected, will be discussed and numerous open problems will be proffered. This is joint work with Danielle Ulrich.

Sven Manthe (University of Bonn)

The Borel monadic theory of order is decidable

Abstract: The monadic second-order theory of $(\mathbb{N}, <)$, S1S, is decidable (it essentially describes ω -automata). Undecidability of the monadic theory of $(\mathbb{R}, <)$ was proven by Shelah. Previously, Rabin proved decidability if the monadic quantifier is restricted to $F\sigma$ -sets. We discuss decidability for Borel sets. Moreover, the Boolean combinations of $F\sigma$ -sets form an elementary substructure. Under determinacy hypotheses, the proof extends to larger classes of sets.

Jana Marikova (University of Vienna)

A definable Tarski's theorem

Abstract: A theorem by Tarski links the existence of certain invariant measures with the non-existence of paradoxical decompositions. We show that in the definable setting, Tarski's theorem can be strengthened. The proof goes via establishing an approximate cancellation law that holds in any model theoretic structure.

Russell Miller (The City University of New York)

Computability questions about infinite Galois groups

Abstract: For an infinite algebraic field extension E/F , Calvert, Harizanov, and Shlapentokh described the automorphisms of E over F as the set of paths through a tree. Under some basic computability assumptions about E and F , this tree is computable, and composition and inversion of the paths are both computable by Turing functionals. Thus we have an effective presentation of the Galois group $\text{Gal}(E/F)$, despite its potentially-continuum size. For finite fields, this presentation of the absolute Galois group is extremely nice, indeed as close to being a decidable structure as one can get in this cardinality. We will explain the notion of “tree-decidability,” developed by Block and the speaker, that makes this rigorous. Predictably, the absolute Galois group of \mathbb{Q} is nastier: we will quantify this nastiness in a few specific ways – one of them joint with Kundu – and pose several open questions about exactly how bad it gets.

Francoise Point (University of Mons)

Expansions of real-closed fields, expansions of Presburger arithmetic and models of open induction

Abstract: I will review classes of "tame" such expansions and their connection via the additive reduct of integer parts of real-closed fields. This is a joint (ongoing) work with Paola D'Aquino.

Bjon Poonen (MIT)

Cohomological obstructions to rational points

Abstract: It is almost a true statement that the only known way to prove that a variety over a number field has no rational point is to exhibit a cohomological obstruction. I will briefly survey a few things that are known about the Brauer-Manin obstruction, the descent obstruction, and related obstructions.

Alexandra Shlapentokh (East Carolina University)

Elliptic curves and definability over big rings.

Abstract: The recent rank stability results have implications for definability over big subrings of number fields and the number fields themselves. We survey these implications.

Keshav Srinivasan (The George Washington University)

Cohesive Powers of Algebraic Number Fields

Abstract: We consider a computability-theoretic ultrapower construction for structures. We start with a computable structure, and consider its countable ultrapower over a cohesive set of natural numbers. A cohesive set is an infinite set of natural numbers that is indecomposable with respect to computably enumerable sets. It plays the role of an ultrafilter, and the elements of a cohesive power are the equivalence classes of certain partial computable functions. Thus, unlike many classical ultrapowers, a cohesive power is a countable structure. We focus on the cohesive powers of fields, specifically number fields. We analyze the algebraic properties of these cohesive powers, and determine just how far their first-order theory is from the original field, utilizing the recent breakthrough concerning Hilbert's Tenth Problem for rings of integers of number fields.

Silvain Rideau-Kikuchi (CNRS, École normale supérieure PSL)

Multi topological fields, approximations and NTP2

Abstract: The striking resemblance between the behaviour of pseudo-algebraically closed, pseudo real closed and pseudo p-adically fields has lead to numerous attempts at describing their properties in a unified manner. In this talk I will present one of these attempts: the class of pseudo-T-closed fields, where T is an enriched theory of fields. These fields verify a « local-global » principle with respect to models of T for the existence of points on varieties. Although it very much resembles previous such attempts, our approach is more model theoretic in flavour, both in its presentation and in the results we aim for. The first result I would like to present is an approximation result, generalising a result of Kollar on PAC fields, respectively Johnson on henselian fields. This result can be rephrased as the fact that existential closeness in certain topological enrichments come for free from existential closeness as a

field. The second result is a (model theoretic) classification result for bounded pseudo-T-closed fields, in the guise of the computation of their burden. One of the striking consequence of these two results is that a bounded perfect PAC field with n independent valuations has burden n and, in particular, is NTP2.

(Joint work with S. Montenegro)
