"Summer School on Probabilistic methods in quantum field theory"

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organized by
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Abstracts

Morris Ang (University of California San Diego)

Solvability of Schramm-Loewner Evolution via Liouville Quantum Gravity

Abstract: Schramm-Loewner evolution (SLE) is a random planar curve arising as the scaling limit of interfaces in critical statistical physics models such as percolation and the Ising model. Remarkably, SLE also describes the interface in the conformal welding of Liouville quantum gravity (LQG) surfaces. This mini-course explores the rich interplay between SLE, LQG, and conformal field theory (CFT). We will derive exact identities linking SLE to CFTs with central charge $c < 1$, and in particular show that a three-point correlation function of SLE agrees with the imaginary DOZZ formula from CFT.

Philémon Bordereau (École Polytechnique Fédérale de Lausanne (EPFL))

SLE and its partition function in multiply connected domains

Abstract: Schramm-Loewner Evolution SLE_κ were introduced by Schramm as the unique family of laws on curves in simply connected domains satisfying conformal invariance and a Markov property. However, in multiply connected domains, there are additional degrees of freedom, and these two properties alone do not uniquely determine SLE_κ.

Lawler suggested imposing the additional requirement of the so-called restriction property to obtain a (non-constructive) characterization. Nevertheless, the mass — or partition function — of the resulting measure is not a priori guaranteed to be finite.

In this talk, we will review two explicit constructions of SLE_κ in multiply connected domains, for different values of κ and various topological cases, from which the partition function can be determined and is finite. Based on joint work with J. Aru.

Harini Desiraju (University of Sydney)

Exploring Probability and CFT via Integrable Structures

Abstract: In this talk, I will outline two nice intersections between the fields mentioned above. The first comes form the developments in the so-called Painlevé/CFT correspondence in the past decade. The second one is the existing relations between SLE curves and integrability, which have already been explored in the literature over a decade ago.

Christophe Garban (Université Lyon 1)

The Berezinskii-Kosterlitz-Thouless (BKT) phase transition

Abstract: One of the main goals of statistical physics is to study how spins displayed along the lattice $\mathbb{Z}^d$ interact together and fluctuate as the temperature changes. When the spins belong to a discrete set (which is the case for example in the celebrated Ising model, where spins σ_x take values in $\{-1, +1\}$) the nature of the phase transitions which arise as one varies the temperature is now rather well understood. When the spins belong instead to a continuous space (for example the unit circle S^1 for the so-called XY model, the unit sphere S^2 for the classical Heisenberg model etc.), the nature of the phase transitions differs drastically from the discrete symmetry setting. The case where the (continuous) symmetry is non-Abelian is even more mysterious (especially when $d = 2$) than the Abelian case. In the latter case, Berezinskii, Kosterlitz and Thouless have predicted in the 70's that these spins systems undergo a new type of phase transition in $d = 2$ — now called the BKT phase transition — which is caused by a change of behaviour of certain monodromies called "vortices".
In this course, I will introduce the intriguing BKT phase transition, explain the key ideas behind recent proofs of its existence, and discuss some of the latest results.

Malin Palö Forrström (Gothenburg University)

An introduction to lattice gauge theories

Abstract: Lattice gauge theories are higher dimensional analogs of famous models such as the Ising model, the XY model, and the clock model. At the same time, they are extremely well-studied models in the physics literature as they are discretizations of the Yang-Mills model in physics, which describes the standard model. In this mini-course, we will introduce lattice gauge theories, discuss and deduce some of their properties, and contrast this to the behavior of, e.g., the Ising model.

Levi Haunschmid-Sibitz (KTH Royal Institute of Technology)

Near-critical dimers and massive SLE_2

Abstract: The uniform dimer model is a classical model from statistical mechanics and one of the few models where conformal invariance has been established. We consider an near-critical weighted version of this model and connect it via the Temperley Bijection and Wilson's Algorithm to a loop-erased random walk. The scaling limit of this walk is (a generalization of) massive SLE_2 as constructed by Markarov and Smirnov and might be of independent interest. In the talk after sketching the connection between the dimer model and the loop-erased random walk, I will focus on this walk and its scaling limit. First I will present some exact Girsanov identities that help connect a random walk with mass with a random walk with drift, and then I will show some of the techniques and difficulties that go into defining the continuum limit.

Liam Hughes (Aalto University)

Embeddability of Liouville quantum gravity metrics

Abstract: Introduced by Polyakov in the 1980s, Liouville quantum gravity (LQG) is in some sense the canonical model of a random fractal Riemannian surface. LQG can be defined as a path integral over fields corresponding to the Liouville action, or equivalently as a random metric measure space that turns out to describe the scaling limit of a host of two-dimensional discrete objects. In particular, certain discrete conformal embeddings of random planar maps converge to canonical (up to conformal reparametrization) embeddings of LQG surfaces into 2D Euclidean space. Though one might expect these metric embeddings to retain some vestige of conformality, in fact no embedding of an LQG surface into $\mathbb{R}^n$ can be quasisymmetric. This generalizes a result of Troscheit in the special case of $\sqrt{8/3}$-LQG (corresponding to uniform random planar maps). Time permitting, I will also discuss future directions in the study of metric embeddability for LQG.

Aleksandra Korzhenkova (École polytechnique fédérale de Lausanne (EPFL))

Entropic Repulsion of Gaussian Free Field by an Interval

Abstract: Motivated by understanding the behavior of spin $O(N)$ models at low temperature, we study the law of the discrete vector-valued 2D Gaussian Free Field conditioned to avoid an N-dimensional ball. In this talk, I will explain the connection between these two types of objects and focus on the case of spin dimension $N = 1$. In this case, the phenomenon of entropic repulsion emerges, along with an ordering of the signs of the conditioned field. I will sketch the proof of this behavior. Joint work with A. Sepúlveda.

Antti Kupiainen (University of Helsinki)

Probabilistic Conformal Field Theory

Abstract: Conformal Field Theories (CFT) are believed to describe the universal behaviour of physical systems at a second order phase transition point. They also play a central role in Quantum Field Theories of fundamental physics by describing their properties in the limits of small and large length scales. In two dimensions they are believed to have a rich mathematical structure uncovered by the physicists Belavin, Polyakov and Zamolodchicov in 1983 with deep impacts in representation theory and geometry. However their rigorous mathematical foundations have remained a matter of debate. The lectures aim to explain a probabilistic approach to CFT based on their path integral formulation and how this can be connected to Graeme Segal's geometric approach to conformal bootstrap, an axiomatic approach to CFT. We discuss in particular two prominent CFTs, the Liouville CFT that plays a central role in Liouville Quantum Gravity and the theory of random surfaces and the Wess-Zumino-Witten models that have rich representation theoretical content.

Irene Ayuso Ventura (Durham University)

The random cluster model on trees and tree recursions

Abstract: The study of certain statistical mechanics models on trees can sometimes be reduced to the study of a "simple" recursion, as is the case for the random cluster model. It turns out that when this recursion is concave, it can be compared to that of effective conductances (potentially nonlinear)

between the root and the leaves of the tree. In collaboration with Quentin Berger (LAGA), we estimated the precise asymptotic behavior of nonlinear conductances on Galton-Watson trees, which allowed us to obtain detailed information about the random cluster model on random trees.

Baojun Wu (Peking University)

Integrability of Conformal Loop Ensemble: Imaginary DOZZ Formula and Beyond

Abstract: The scaling limit of the probability that n points are on the same cluster for 2D critical percolation is believed to be governed by a conformal field theory (CFT). Although this is not fully understood, Delfino and Viti made a remarkable prediction on the exact value of a properly normalized three-point probability from the exact S-matrix. It is expressed in terms of the imaginary DOZZ formula. Later, similar conjectures were made for scaling limits of random cluster models and O(n) loop models, combining both integrable structure of discrete model as well as the bootstrap hypothesis, representing certain three-point observables in terms of the imaginary DOZZ formula and its variants. Since the scaling limits of these models can be described by the conformal loop ensemble (CLE), such conjectures can be formulated as exact statements on CLE observables. This talk explains the derivation of the above three-point functions via Liouville quantum gravity.

This is based on the joint work with Morris Ang (UC San Diego), Gefei Cai (BICMR), and Xin Sun (BICMR).
