HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

"Random geometry and its connections to QFT"

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organized by
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Abstracts

Nathanaël Berestycki (Universität Wien)

Massive holomorphicity in the near-critical dimer model

Abstract: We consider the dimer model on isoradial graphs, in the near-critical scaling regime and with Temperleyan boundary conditions. In previous joint work with Levi Haunschmid-Sibitz we proved (for the square and hexagonal lattices) convergence of branches in the associated Temperleyan tree to the so-called massive SLE_2 of Makarov and Smirnov. We also stated a precise conjecture that the limiting height function is a specific variant of the sine-Gordon model (from quantum field theory) at its free fermion point. This conjecture is currently being proved in joint work with Scott Mason and Lucas Rey. I will describe some of our results in this direction.

Avelio Sepúlveda (Universidad de Chile)

Excursion decompositions in statistical physics

Abstract: In this talk, I will discuss excursion decompositions in various models of statistical physics. By an excursion decomposition, we mean a representation of a field as a sum $\sum_n \mu_n \sigma_n$, where the μ_n are positive measures with disjoint supports and the σ_n are i.i.d. fair signs. The models to be discussed are both the discrete and continuous versions of the random walk, Ising model, Gaussian free field and the XOR-Ising model. This talk is based on joint work with Tomás Alcalde, Juhan Aru, and Titus Lupu.

Oren Louidor (Technion Israel Institute of Technology)

Gaussian Free Field on the Tree Subject to a Hard Wall

Abstract: We study the discrete Gaussian free field on the binary tree when all leaves are conditioned to be positive. We obtain sharp asymptotics for the probability of this "hard-wall constraint" event, and identify the repulsion profile followed by the field in order to achieve it. We then provide estimates for the mean, fluctuations and covariances of the field under the conditioning, which show that in the first log-many generations the field is super-exponentially tight around its mean. These results are then used to obtain a comprehensive, sharp asymptotic description of the law of the field under

this conditioning. We provide asymptotics for both local statistics, namely the (conditional) law of the field in a neighborhood of a vertex, as well as global statistics, including the (conditional) law of the minimum, maximum, empirical population mean and all subcritical exponential martingales. We conclude that, even in a local sense, the recentered repelled field is asymptotically not the unconditional field, thereby resolving an open question of Velenik from 2006, albeit in the analogous case of the tree. Joint work with Maximilian Fels (Technion) and Lisa Hartung (Mainz).

Guillaume Baverez (Aix-Marseille University)

Integration by parts in random conformal geometry and applications

Abstract: The Gaussian free field and Schramm-Loewner evolutions are prominent examples of stochastic processes exhibiting conformal invariance. In these talks, we will discuss the variational formulas of these measures under local conformal transformations, and deduce some integration by parts formulas, which can be neatly phrased as representations of the Virasoro algebra on the L2-space of the measures. This algebraic structure has nice probabilistic consequences, including a characterisation of SLE and a new approach to the conformal welding of quantum surfaces.

Antoine Jego (Paris Dauphine)

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Yilin Wang (IHES)

Brownian loop measure on Riemann surfaces and length spectra

Abstract: Brownian loop measure is a sigma-finite measure on Brownian-type loops on the Riemann sphere, which satisfies conformal invariance and restriction property. It was introduced by Lawler and Werner and has a tight link to random curves appearing in 2D conformally invariant systems: Schramm-Loewner evolution (SLE), Conformal loop ensemble (CLE), etc.
We consider its generalization to an arbitrary Riemann surface and show that the lengths of closed geodesics for the constant curvature metric are also encoded in the Brownian loop measure. This gives a tool to study the length spectra of Riemann surfaces. In particular, we obtain a new identity between the length spectrum of a surface and that of the same surface with an arbitrary number of additional cusps. We also express the determinant of Laplacian of a compact hyperbolic surface as the total mass of Brownian loop measure renormalized according to the length spectrum. This is based on a joint work with Yuhao Xue (IHES).

Semyon Klevtsov (Université de Strasbourg)

Quantum hall states on hyperbolic surfaces

Abstract: One major application of free fields and their conformal blocks is that they give trial many-body wave functions in quantum Hall effect, notably Laughlin states and their generalizations. We will construct Laughlin on compact higher genus surfaces using free fields and also talk about some recent results for integer QHE state on the hyperbolic cylinder, based on work to appear with P. Wiegmann.

Shirshendu Ganguly (UC Berkeley)

Critical last passage percolation

Abstract: Last passage percolation (LPP) is a model of random geometry where the main observable is a directed path evolving in a random environment. When the environment distribution has light tails and a fast decay of correlation, the random fluctuations of LPP are predicted to be explained by the Kardar–Parisi–Zhang (KPZ) universality theory. However, the KPZ theory is not expected to apply in many natural settings, such as "critical" environments exhibiting a hierarchical, fractal-like structure which should give rise to a fluctuation theory featuring logarithmic corrections with novel critical exponents. Predictions for these exponents are missing, even from the physics literature. In recent joint work with Victor Ginsburg and Kyeongsik Nam we initiated the study of LPP in hierarchical environments, developing a framework based on multi-scale analysis and obtaining bounds on critical exponents for two canonical examples: an i.i.d. environment with critical power-law tails, and a hierarchical approximation of the two-dimensional Gaussian Free Field. In this talk I will discuss these results and, time permitting, will also touch upon ongoing work with Victor Ginsburg and Kaihao Jing exploring connections to fractal percolation as well as related polymer models.

Ajay Chandra (Imperial College)

Stochastic quantization beyond bosons

Abstract: In this talk I will describe some recent progress on developing stochastic analysis tools for studying Langevin dynamics for quantum field theories that sit outside of the scope of classical probability theory, such as fermionic field theories. This is based on joint work with Martin Hairer and Martin Peev.

Ofer Zeitouni (Weizmann Institute of Science)

Extremes for the two dimensional Liouville model

Abstract: We establish a strong coupling between the Liouville model and the Gaussian free field on the two dimensional torus, which bounds the difference of the two fields uniformly by a random constant. As an application we show that the maximum of the Liouville field converges in distribution to a randomly shifted Gumbel distribution. Joint work with Michael Hofstteter.

Hao Shen (University of Wisconsin - Madison)

Makeenko-Migdal equations for 2D Yang-Mills: from lattice to continuum

Abstract: An important type of observables in the Yang-Mills model is the Wilson loops. They satisfy useful recursions, called the Makeenko-Migdal equations, also called the loop equations, which are essentially integration by parts or Dyson-Schwinger equations in the context of Yang-Mills. They

can be rigorously formulated and proved on lattices of arbitrary dimensions (by Chatterjee, Cao-Park-Sheffield, etc.), or in 2D continuum (by Levy, Dahlqvist, Driver-Hall-Kemp, etc). We prove that in 2D, the lattice Makeenko-Migdal equations converge to the continuum ones in an appropriate sense. Based on joint work with S.Smith and R.Zhu.

Fredrik Viklund (KTH)

Free energy of confined planar Coulomb gases

Abstract: Consider a Coulomb gas of charged particles confined to a set in the complex plane. I will discuss the following question: How does the asymptotic expansion of the free energy depend on the geometry of the set, as the number of particles tends to infinity? When the set is a Jordan domain, curve or arc, this problem is related to Grunsky- operators associated to the set, revealing a close connection to the Loewner energy and other interesting domain functionals. Based joint works with K. Courteaut (NYU) and K. Johansson (KTH).

Marek Biskup (UCLA)

Phase diagram and extremal properties of the hierarchical integer-valued GFF

Abstract: I will review recent progress, obtained jointly with my student H. Huang, on the hierarchical integer-valued GFF, a.k.a., the DG model or Villain Coulomb gas model. Specifically, I will describe the asymptotic form of the covariance and the characteristic function of the field at inverse temperatures below, at and slightly above a critical value where both of these quantities undergo a phase transition. Below the critical inverse temperature, where the field is asymptotically close to continuum-valued GFF, I will give a description of the scaling limit of the absolute maximum and the associated extremal process. The main technique of proofs is renormalization which, although much easier in the hierarchical context than on a lattice, is still quite difficult to control near, let alone beyond, the critical point.

Léonie Papon (Durham University)

On the scaling limit of interfaces in the critical planar Ising model perturbed by a (random or not) magnetic field

Abstract: In this talk, I will present some recent results on the scaling limit of interfaces in the critical planar Ising model perturbed by a magnetic field. I will first consider the case when the field is a deterministic function. In this case, I will show that in the so-called near-critical regime and when the Ising model has Dobrushin boundary conditions, the interface separating $+1$ and -1 spins has a scaling limit whose law is conformally covariant and absolutely continuous with respect to SLE_3. Its limiting law is a massive version of SLE_3 in the sense of Makarov and Smirnov. I will also discuss the scaling limit of this interface when the magnetic perturbation is not near-critical. In the second part of the talk, I will look at the case when the magnetic field is given by a collection of iid centered Gaussian random variables, one for each vertex. In this setting, in the near-critical regime, I will show that almost surely in the disorder, the scaling limit of the quenched law of the ± 1 interface is absolutely continuous with respect to SLE_3. I will then show that this contrasts with the scaling limit of the quenched law of the collection of nested spin loops, which turns out to be almost surely singular with respect to nested CLE_3. This also contrasts with the deterministic case where it is known that in the near-critical regime, any subsequential limit of the collection of nested spin loops is absolutely continuous with respect to nested CLE_3.
