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 Workshop

# “Integrable QFT: conformal bootstrap, bosonization, near-critical perturbations, and Coulomb gas”

June 23 -27, 2025

organized by

Roland Bauerschmidt, Eveliina Peltola, Ellen Powell, Rémi Rhodes, Eero Saksman

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## Abstracts

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**Kalle Kytölä** (University of Helsinki)

### Local fields in lattice models and conformal field theories

**Abstract:** In the 1980’s, physicists conjectured that scaling limits of probabilistic lattice models at criticality should be described by conformal field theories (CFT). The state space of a CFT is also its space of local fields, and the observable quantities are correlation functions of local fields. A natural analogue of local fields in probabilistic lattice models is random variables built locally from the basic degrees of freedom of the model. Surprisingly, in certain two-dimensional models, discrete complex analysis can be used to equip spaces of lattice model local fields with the main algebraic structure of CFTs: a representation of the Virasoro algebra. This opens up the possibility of full and structured correspondence between the space of local fields of a CFT and of its lattice discretization. For the model of discrete Gaussian Free Field (dGFF), we proved that the space of local fields of the gradient of the dGFF is isomorphic to a Fock space of a free bosonic CFT, and that correlations of the dGFF local fields converge in the scaling limit to correlation functions of that CFT, when appropriately renormalized according to the eigenvalues of the Virasoro generators corresponding to the energy in CFT. Partial results have been obtained and progress is being made towards analogous complete CFT description of the Ising model.

The talk is based on joint works with Clément Hongler (EPFL Lausanne), Fredrik Viklund (KTH Stockholm), David Adame-Carrillo, Delara Behzad, Dmitry Chelkak (Univ. Michigan), and Konstantin Izyurov (Univ. Helsinki)

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**Hao Wu** (Tsinghua University)

### Commutation relation and BPZ equations

**Abstract:** In 1999, O. Schramm introduced Schramm-Loewner evolution (SLE) as a non-self-crossing random curve driven by a multiple of Brownian motion using Loewner’s transform. This definition is motivated by a quest to mathematically describe the random interfaces in 2D critical lattice models, which satisfy the conformal invariance and domain Markov property. In this talk, we will consider the law of multiple curves with conformal invariance and domain Markov property, following the framework of Dubédat’s commutation relation.

To solve Dubédat's commutation relation, one needs to classify solutions to Belavin-Polyakov-Zamolodchikov (BPZ) equations. There are various works related to the classification of solutions to BPZ equations in the chordal setting, which we will review in this talk. BPZ equations in the radial setting are less explored. We will present recent results about classification of BPZ equations in the radial setting and explain its connection to the random-cluster model and Gaussian free field.

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**Tom Alberts** (University of Utah)

### **Dubédat Screening and Level Two Non-Degeneracy**

**Abstract:** Dubédat screening refers to a conformal field theoretic method for solving BPZ equations, a system of PDEs whose solutions describe multiple chordal SLE curves in a simply connected domain. This talk explains how the solution introduces a level two non-degenerate vertex exponential into the system, and how this leads to a loss of a martingale observable property for the evolution of a field under the corresponding SLE flow. I will then present a simple fix to the Dubédat screening system.

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**Alessandro Giuliani** (University of Rome 3)

### **Anomalous exponents and scaling operators in a 3D fermionic $\phi^4$ theory**

**Abstract:** We consider a model of 3D symplectic fermions with fractional kinetic term, compute its field and density critical exponents, and prove that the scaling dimension of the density operator is anomalous and analytic in the distance  $\epsilon$  from the non-interacting reference Gaussian theory. We define scaling operators associated with the field and the density and prove scale invariance of their response functions. Connections with a CFT description of the model will be discussed. Based on joint work with Vieri Mastropietro, Slava Rychkov and Giuseppe Scola <https://arxiv.org/abs/2404.14904>.

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**Masha Gordina** (University of Connecticut)

### **Fermionic stochastic analysis**

**Abstract:** I will talk about stochastic analysis of Grassmann random variables suitable for the stochastic quantization of Euclidean fermionic quantum field theories. The idea of using Grassmann random variables for fermionic systems goes back to Osterwalder and Schrader, and to implement it we used tools from non-commutative analysis. I will describe some of the stochastic analytical tools such as Grassmann Brownian motion and non-commutative  $L^p$  spaces, and mention some of the applications. Based on the joint work with F. De Vecchi, L. Fresta and M. Gubinelli.

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**Wei Wu** (NYU Shanghai)

### **Massless phases for the Villain model in $d \geq 3$**

**Abstract:** The XY and the Villain models are models which exhibit the celebrated Kosterlitz-Thouless phase transitions in two dimensions. The spin wave conjecture, originally proposed by Dyson and by Mermin and Wagner, predicts that at low temperature, spin correlations of these models are closely related to Gaussian free fields. I will review the historical background and present some recent progress for the Villain model in  $d \geq 3$ . Based on the joint work with Paul Dario (CNRS).

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**Xin Sun** (Peking University)

### **Application of Liouville Quantum Gravity to 2D Percolation**

**Abstract:** I will use 2D percolation as an example to demonstrate how Liouville quantum gravity (LQG) is used in the recent progress on the exact solvability in 2D statistical physics. Comparing to previous applications of quantum gravity in statistical physics based on the Kniznik-Polyakov-Zamolodchikov (KPZ) relation, the crucial novelty of the recent applications is the interplay between the exact solvability in Liouville conformal field theory and the coupling theory of LQG and SLE.

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**Colin Guillarmou** (Paris-Saclay University)

### **Probabilistic construction of $H^3$ Wess-Zumino-Witten CFT**

**Abstract:** We'll explain how to produce a probabilistic construction of the WZW model with values in the hyperbolic 3-space  $H^3 = SL(2, C)/SU(2)$ , including in the gauged case, on compact Riemann surfaces. We prove a conjecture from physics that the correlation functions can be expressed in terms of Liouville correlation functions, in a quite non-trivial way. Joint work with Kupiainen and Rhodes.

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**Yi-Zhi Huang** (Rutgers University)

### **The representation theory of vertex operator algebras, conformal bootstrap and Segal's axioms**

**Abstract:** The representation theory of vertex operator algebras is one of the most successful approaches to the construction and study of conformal field theories. In this approach, starting with some basic algebraic data satisfying suitable finiteness conditions, we construct spaces of states and correlation functions and prove the basic assumptions of conformal bootstrap and Segal's axioms for conformal field theories.

In this talk, I will review the results on conformal bootstrap and Segal's axioms obtained using the representation theory of vertex operator algebras. I will also discuss the possible implications of the construction of Liouville conformal field theory for the representation theory of vertex operator algebras in the case that the finiteness conditions are not satisfied.

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**Nikolai Barashkov** (Max Planck Institute Leipzig)

### **Small deviations of Gaussian multiplicative chaos and the free energy of the two-dimensional massless Sinh-Gordon model**

**Abstract:** We give a bound on the probability that the total mass of Gaussian multiplicative chaos measure obtained from a Gaussian field with zero spatial average, is small. We also give the probabilistic path integral formulation of the massless Sinh-Gordon model on a torus of side length  $R$ , and study its partition function as  $R$  tends to infinity. We apply the small deviation bounds for Gaussian multiplicative chaos to obtain lower and upper bounds for the logarithm of the partition function, leading to the existence of a non-zero and finite subsequential infinite volume limit for the free energy.

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**Eric Schippers** (University of Manitoba)

## A geometric interpretation of the Grunsky operator

**Abstract:** The Grunsky operator is a matrix associated to a conformal map of the disk into the Riemann sphere, which under sufficient conditions on the mapping can be treated as a bounded operator. It arises classically in potential theory and extremal problems for conformal maps, for example. The Grunsky operator also surfaces in Weil-Petersson Teichmüller theory, representation theory, and conformal field theory, and is enjoying a surge of interest. It has many (roughly) equivalent formulations, but its interpretation is somewhat elusive. This talk focusses on results establishing a particular geometric meaning of the Grunsky operator in terms of boundary values of functions or one-forms and polarizations. We give generalizations to the case of finitely many conformal maps into a compact Riemann surface.

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**André Henriques** (University of Oxford)

## The functorial formalism for chiral 2d CFT: definition, and methods for constructing examples

**Abstract:** I will provide a gentle explanation to the functorial approach to 2d CFT, with an emphasis on the crucial difference between full CFT and chiral CFT. I will sketch some methods that I hope to use one day to construct examples of chiral CFTs in the functorial formalism.

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**Ilya Losev** (University of Cambridge)

## Probabilistic Schwarzian Field Theory

**Abstract:** Schwarzian Theory is a quantum field theory which has attracted a lot of attention in the physics literature in the context of two-dimensional quantum gravity, black holes and AdS/CFT correspondence. It is predicted to be universal and arise in many systems with emerging conformal symmetry, most notably in Sachdev–Ye–Kitaev random matrix model and Jackie–Teitelboim gravity. In this talk we will discuss our recent progress on developing rigorous mathematical foundations of the Schwarzian Field Theory, including rigorous construction of the corresponding measure, calculation of both the partition function and a natural class of correlation functions, and a large deviation principle.

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**Baptiste Cercle** (EPFL Lausanne)

## Towards solvability of Toda Conformal Field Theories

**Abstract:** Toda Conformal Field Theories (CFTs) form a family of two-dimensional CFTs that generalize the celebrated Liouville theory. One of the defining features of Toda CFTs is that they enjoy, in addition to conformal invariance, an enhanced level of symmetry encoded by W-algebras. Besides they can be rigorously defined within a probabilistic framework that involves (vectorial) Gaussian Free Fields and Gaussian Multiplicative Chaos.

During this talk I will review some recent progress and current works related to solvability of these models. As we will see a key point is the translation of results coming from representation theory of W-algebras into actual statements for Toda correlation functions, which in turn provides a systematic method to address the Mukhin-Varchenko conjecture for Selberg-type integrals. This is based on a series of work with Nathan Huguenin.

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