Report on the Trimester Program

Metric Analysis

January 13 - April 17, 2025

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Topics

The Trimester Program on *Metric Analysis*, focused on the development of analytic tools for studying geometric structures in general metric spaces, extending beyond the classical setting of Riemannian manifolds.

Metric analysis has emerged as a central area of modern mathematics, connecting techniques from analysis, geometry, and topology. It provides a framework for understanding spaces that may lack smooth structure but still exhibit rich geometric and analytic behavior. In particular, the study of invariants such as filling volumes, norms, and energies plays a crucial role in capturing the large-scale and fine geometric properties of such spaces.

The subject interacts with a wide range of mathematical disciplines, including: analysis on metric spaces, geometric group theory, differential geometry, higher Teichmüller theory, and low-dimensional topology.

These connections highlight both the versatility and the complexity of the field. Researchers from different backgrounds contribute diverse perspectives, making it both a dynamic and challenging area to synthesize into a unified framework.

Goals

The primary goal of the Trimester Program was to advance the field of metric analysis by fostering interaction between researchers working on its various aspects.

A central objective was the refinement and development of analytic tools applicable to non-smooth geometric settings. This includes the study of:

- filling invariants and isoperimetric inequalities,

- energy functionals and variational methods,
- the structure of metric and metric-measure spaces.

The program aimed to promote the exchange of ideas across subfields and to encourage collaboration between experts in different areas of geometry and analysis. Informal discussions, seminars, and collaborative working groups formed an integral part of the program, allowing participants to explore new approaches and identify common themes.

Another important component was the inclusion of early-career researchers. Introductory activities, such as a winter school *Analysis and geometry on groups and spaces*, provided PhD students and young researchers with exposure to current developments and open problems in the field.

Organization

The Trimester Program was structured around several major scientific events, complemented by a regular seminar series and ongoing research activities.
1. An introductory winter school "Analysis and geometry on groups and spaces" was held January 27–31, 2025. The winter school provided an introduction to key concepts and techniques in metric analysis, with a focus on making advanced topics accessible to graduate students and early-career researchers.

The lecture courses were: "An Introduction to Systolic Geometry"Panos Papasoglu, "Ricci curvature and fundamental group" by Daniele Semola, "Quantitative Topology and Geometric Inequalities" by Regina Rotman and "An introduction to rectifiability in metric spaces" by David Bate.

These courses established a common foundation for participants and introduced central tools and problems that were further explored throughout the trimester.
2. An international conference "Differential geometry beyond Riemannian manifolds", March 24–28, 2025.

The conference brought together leading experts to present recent developments in both smooth and non-smooth geometric settings, with a particular emphasis on metric measure spaces and synthetic curvature bounds.

The speakers included: Richard Bamler, Christine Breiner, Cornelia Drutu, Giada Franz, Ailana Fraser, Bernhard Hanke, Bruce Kleiner, Urs Lang, Alexander Lytchak, Fedor Manin, Damaris Meier, Chikako Mese, Alexander Nabutovsky, Pierre Pansu, Melanie Rupflin, Roman Sauer, Antoine Song.

The talks reflected the breadth of modern geometric analysis and highlighted connections between classical differential geometry and its metric generalizations.

3. Felix Klein Lectures, held March 31 – April 11, 2025.

A central highlight of the program was the Felix Klein Lecture series, delivered by André Neves, one of the leading experts in geometric analysis.

The lecture series, titled "Recent Progress on the theory of minimal surfaces", consisted of six lectures. These lectures provided an in-depth overview of recent breakthroughs in the theory of minimal surfaces, connecting variational methods, geometry, and topology. The series was designed to be accessible to a broad mathematical audience while also presenting cutting-edge research developments.

4. In addition to the main events, a regular seminar series was held throughout the trimester, featuring talks by invited researchers and participants. A particular highlight was the inclusion of the GNOSC seminar – an online seminar series initiated by Gromov and collaborators – within our program. As part of this collaboration, Roman Sauer gave an in-person talk that was simultaneously live-streamed.

The seminar series also provided a platform for ongoing research discussions, allowing participants to engage with current developments in a more informal and interactive setting.

Results

The following list contains preprints and published articles by March 2026 which mention the trimester program in the acknowledgements.

1. T. Foertsch, A. Lytchak, E. Soultanis,
   Hilbert space factor of metric spaces, arXiv:2503.00864

2. J. Block, F. Manin, S. Weinberger,
   Persistent homology of function spaces, arXiv:2505.16907

3. J. Seidel,
   An Invariant for Triple-Point-Free Immersed Spheres, arXiv:2506.21130

4. E. Mäder-Baumdicker, J. Seidel,
   The Willmore Energy Landscape of Spheres and Avoidable Singularities of the Willmore Flow, arXiv:2506.23359

5. V. Ryborz,
The infinitesimal structure of manifolds with non-continuous Riemannian metrics, arXiv:2507.14726

6. U. Lang, S. Stadler, D. Urech,
Isoperimetric inequalities in Hadamard spaces of asymptotic rank two, arXiv:2506.04882

7. D. Meier, N. Vikman, S. Wenger,
Energy minimizing harmonic 2-spheres in metric spaces, arXiv:2503.08553

8. A. Mondino, C. Sämann,
Lorentzian Gromov-Hausdorff convergence and pre-compactness, arXiv:2504.10380

9. A. Cucinotta, A. Mondino,
On manifolds with almost non-negative Ricci curvature and integrally-positive k-th scalar curvature, arXiv:2504.06865

10. M. Magnabosco, A. Mondino, T. Rossi,
On the rectifiability of CD(K, N) and MCP(K, N) spaces with unique tangents, arXiv:2505.01151

11. F. Cavalletti, D. Manini, A. Mondino,
On the geometry of synthetic null hypersurfaces, arXiv:2506.04934

12. A. Cucinotta, M. Magnabosco, D. Semola,
New Topological Restrictions For Spaces With Nonnegative Ricci Curvature, arXiv:2601.14231

13. D. Maximo, Y. Liokumovich, R. Rotman,
Length of a closed geodesic in 3-manifolds of positive scalar curvature, arXiv:2504.05459

14. B. Staffa,
Weyl law for 1-cycles, arXiv:2410.23192

15. B. Staffa,
Parametric coarea inequality for 1-cycles, arXiv:2410.23195

16. T. Talipov,
Besicovitch-type inequality for closed geodesics on 2-dimensional spheres, arXiv:2412.02028