Short research presentations

August 21, 13:30-16:00

13:30 Heiko Dietrich

The group isomorphism problem is easy for almost all group orders

We discuss why the title is true for finite groups in the Cayley table model. Joint work with James Wilson (FOCS 2022).

13:45 Martin Kassabov

Presentations of Simple Groups of Ree type

We show what the finite simple groups of Ree type have presentations with a bounded number of generators and relations independent of the size of the field. A starting point of the argument is the Steinberg presentation which comes from viewing ${}^2G_2(\mathbb{F}_q)$ as a rank 1 group. The key step is reducing the number of relations from q^3 to a bounded number. This uses algebraic geometry over rings with extra endomorphism, and showing that certain varieties have points defined over the finite field $\mathbb{F}_q$. Joint work with Alexander Hulpke, Akos Seress, and James Wilson.

14:00 Gabi Nebe

On the natural involution in group algebras of finite groups

Given a semisimple rational algebra A with an involution ι, what are the conditions for A to be the group algebra $A \cong \mathbf{Q}G$ for some finite group G where $\iota : g \mapsto g^{-1}$ is the natural involution?

We computed invariants of the restriction of the natural involution to direct summands of $\mathbf{Q}G$ for the ATLAS groups up to the Harada–Norton group and for many finite groups of Lie type (joint with Richard Parker, Thomas Breuer, David Schlang and Linda Hoyer).

There are certain surprising facts for adjoint involutions of an invariant quadratic form: For some sporadic groups the quadratic extension of the character field determined by the discriminant of the invariant quadratic forms is not Galois over the rationals.

We never observed square classes having an odd dyadic valuation, motivating Richard Parker to conjecture that these discriminants are always odd. This conjecture is a theorem for solvable groups and for some infinite series of finite groups of Lie type; however we do not have a structural explanation for this phenomenon.

14:15 Tobias Rossman

Enumerating conjugacy classes of graphical groups

Each choice of a (finite, simple, undirected) graph and commutative ring gives rise to an associated graphical group. I will report on recent developments surrounding the symbolic enumeration of conjugacy classes of graphical groups over natural infinite families of finite rings.

14:30-45 Break

14:45 Christopher Voll

Hall-Littlewood polynomials, affine Schubert series, and lattice enumeration

In this talk, I would like you to meet Hall-Littlewood-Schubert series, a new class of multivariate generating functions. Their definition features semistandard Young tableaux and polynomials resembling the classical Hall-Littlewood polynomials. Their intrinsic beauty notwithstanding, Hall-Littlewood-Schubert series have many applications to counting problems in algebra, geometry, and number theory. In my talk the spotlight will be on applications to affine Schubert series. These may be seen as an integral analogue of the Poincaré polynomials enumerating the rational points over finite fields of classical Schubert varieties. The latter parametrize subspaces of a given vector space by the intersection dimensions with a fixed flag of reference. I will explain things from scratch, assuming no familiarity with the advanced technical vocabulary used in this abstract. Joint with Joshua Maglione.

15:00 Christian d'Elbée

Lie methods for omega-categorical Engel groups

A structure (group, Lie algebra, associative algebra, etc.) M is ω-categorical if there is a unique countable model of its first-order theory, up to isomorphism. This model theoretic notion has a dynamical definition: M is omega-categorical if and only if there are only finitely many orbits in the component-wise action of $\mathrm{Aut}(M)$ on the cartesian power M^n, for all natural number n. In 1981, Wilson conjectured that any ω-categorical locally nilpotent p-group is nilpotent. If true, a quite satisfactory decomposition of every ω-categorical group would follow. This conjecture is very much open more than 40 years later.

The analogue statement for Lie algebras (every locally nilpotent omega-categorical Lie algebra is nilpotent) is also open. Both statements can be reformulated as: any omega-categorical Engel group/Lie algebra is nilpotent. As such, those questions are connected to Burnside-type problems and the work of Higman, Kostrikin, Zelmanov, Vaughan-Lee, Traustason, etc. For instance, using a classical result of Zelmanov, the conjecture for Lie algebras is true asymptotically in the following sense: for each n, every n-Engel Lie algebra over $\mathbb{F}_p$ is nilpotent for all but finitely many p. There is a similar statement for groups. The situation for small values of the pair (n, p) is highly characteristic-dependent, for instance, 4-Engel Lie algebras over a field of characteristic p are nilpotent except if $p = 2, 3$ or $p = 5$. I recently proved that omega-categorical n-Engel Lie algebras over a field of characteristic p are nilpotent for $(n, p) = (3, 5)$ and $(n, p) = (4, 3)$. Lie methods can then be used in order to deduce that 5-Engel 5-groups are nilpotent, and I will briefly explain the main ingredients for those results.

15:15 Emmanuel Rauzy

Effective residual finiteness and a question of Higman

Being able to algorithmically list the finite quotients of a finitely generated group provides a way to use algebraic properties (like residual finiteness, subgroup separability, or profinite rigidity), to obtain solution to decision problems (here, the word problem, the membership problem or the isomorphism problems). Recent results of Nies related to computability of profinite groups provide new reasons to explore such questions. I will present results from my PhD about computability of finite quotients of finitely generated group, and a new result obtained with Arman Darbinyan: there is a finitely generated residually finite group with solvable word problem that does not embed in the group of computable permutations of the natural numbers.

15:30 Frank Wagner

Skew Braces of small Morley Rank

A skew brace is a structure which carries two group laws $+$ and $*$, related by a left distributive law $a(b+c) = ab - a + ac$. They were introduced in an attempt to analyze solutions to the set-theoretic Yang–Baxter equation from theoretical physics, but have increasingly been studied for their algebraic properties. I shall present a classification of skew braces of Morley rank at most three. This is joint work with my student Moreno Invitti, as well as Maria Ferrara (Caserta) and Marco Trombetti (Naples).

15:45 Jeroen Schillewaert

Discreteness and freeness of 2-generator subgroups of $SL_2(K)$, where K is a local field

I will give a brief overview of some recent results obtained jointly with Matthew Conder in the non-archimedean case, by my student Ari Markowitz over the reals, and jointly with Alex Elzenaar and Gaven Martin over the complex numbers.