**Introductory School**

# “Geometric Statistics”

**September 14 - 18, 2026**

organized by
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## Abstracts

**Fernando Galaz-García** (Durham University)

### Alexandrov and RCD Spaces: A User’s Introduction

**Abstract:** Alexandrov spaces and RCD spaces provide two complementary frameworks for extending lower curvature bounds beyond the setting of smooth Riemannian manifolds. Alexandrov geometry keeps a synthetic lower bound on sectional curvature through triangle comparison, while the RCD condition combines a synthetic lower Ricci curvature bound with a compatible metric-measure and analytic structure. This introductory lecture series will develop the basic ideas, examples and tools of both theories. We will begin with Alexandrov spaces, discussing triangle comparison, tangent cones, spaces of directions, singular points and Gromov-Hausdorff convergence. We will then turn to metric-measure spaces and introduce the curvature-dimension and RCD conditions through their connections with optimal transport and weak differential calculus. We will also discuss the relation between the two theories. The emphasis will be on examples, geometric intuition, and the main results needed to begin using these spaces.

**Elton Hsu** (Northwestern University)

### Stochastic Analysis on Manifolds

**Abstract:** In this short course, I will discuss basic theory of stochastic analysis on manifolds based on general semimartingale theory. The topics include manifold valued semimartingales, stochastic differential equations on manifolds and diffusion processes on manifolds. Both geometrically invariant and local coordinates formulations will be covered. If time permits, I will also discuss manifold valued martingales and their connection with backward stochastic differential equations.

**Xavier Pennec** (Inria centre at Université Côte d'Azur)

### Geometric Statistics for Computational anatomy

**Abstract:** Computational anatomy focuses on statistically modelling anatomical shapes, such as points, curves, surfaces, or images, to quantify normal and pathological variations in a population. However, these geometric objects naturally live in non-linear spaces, while traditional statistics are designed for Euclidean frameworks. By considering shapes as points of a manifold, one can leverage Riemannian geometric tools to define intrinsic distances, angles, and geodesics (shortest paths), generalising classical geometry to curved spaces like spheres or hyperbolic surfaces.
Key statistical concepts are redefined in this context. For example, the Fréchet mean minimises the sum of squared distances to observations, enabling consistent (weighted) averaging in manifolds. This framework extends image processing algorithms (e.g., interpolation, filtering) to data like Diffusion Tensor Imaging (DTI), where each voxel represents a covariance matrix.
Statistically, estimating means in manifolds reveals curvature-dependent effects: convergence rates vary with curvature, and bias arises from curvature gradients. Principal Component Analysis (PCA) is adapted as tangent PCA (tPCA), maximising explained variance, or Principal Geodesic Analysis (PGA), minimising unexplained variance via geodesic subspaces. Barycentric subspaces generalise affine spaces, forming nested hierarchies for robust dimensionality reduction.
Finally, statistics on deformation groups (e.g., LDDMM) use right-invariant Riemannian metrics, though symmetry limitations exist. Alternatives like Cartan-Schouten connections enable geodesics without distances, supporting efficient diffeomorphism parametrisation. Applications include modelling brain atrophy in Alzheimer's disease.

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**Amy Willis** (University of Washington)

### Asymptotic Statistics on Stratified Spaces

**Abstract:** Central limit theory underpins modern statistical practice. This introductory lecture series will begin by reviewing central limit theory for data objects in Euclidean space, then discuss generalizations to manifold-valued data and data in stratified spaces. We will discuss the implications of positive and negative curvature on estimator behaviour and rates, including the phenomena of smeariness and stickiness. Important examples of non-standard sample and parameter spaces will be given. The discussion will focus on Fréchet means, though related parameters will also be mentioned. The series will emphasize the contrast with Euclidean asymptotics and geometric intuition for the contrast.

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