HAUSDORFF RESEARCH INSTITUTE
FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop

"Optimal approximation spaces for multiscale problems"

February 23 - 27, 2026

organized by
Moritz Hauck, Johan Wärnegård

Abstracts

Lucia Swoboda (Chalmers University of Technology)

Hybridizable discontinuous Galerkin methods for the wave equation on beam network models

Abstract: Network models are used to describe complex structures found in fiber-based materials such as paper, or biological tissues like blood vessels. In these models, fibers are represented as edges connected at nodes, with each fiber modeled as a beam to capture the mechanical behavior of the material. Understanding wave propagation in such networks is important for predicting material response and improving performance in applications such as papermaking. In this talk, I will present a hybridizable discontinuous Galerkin (HDG) method for the wave equation on fiber networks. Through hybridization, the problem can be reformulated as a symmetric positive definite system posed on the network nodes, while maintaining high-order accuracy on the individual fibers. I will outline the main ideas behind the convergence analysis and error estimates, supported by numerical experiments. Finally, I will introduce a two-level overlapping domain decomposition method designed for efficient parallelization of the global system.

Joseph Holten (KIT - Karlsruher Institut für Technologie)

Domain Decomposition Methods for Elastic Wave Propagation

Abstract: This paper considers the numerical solution of the first order formulation of the elastic wave equation posed on a network of timoshenko beams, which are coupled by rigid joint conditions. Such networks arise from modelling the complex, heterogeneous geometries of fiber-based materials like paper. Space-discretization is performed with the hybrid discontinuous galerkin method,resulting in a symmetric positive system of linear equations posed on the nodes of the network.Heterogeneous material coefficients and the resolution of the fiber scale over the material scale result in a very large, very poorly conditioned global system matrix. This requires an implicit time-integrator, the theta scheme was chosen for simplicity. In each timestep, the global system of linear equations is solved iteratively using the preconditioned conjugate gradient method. A two-level delta-overlap additive Schwarz method was employed as the preconditioner using a partition of unity basis constructed via algebraic partitioning of the network. It is proven, that under appropriate homogeneity and multiplicity conditions on the resulting algebraic cover, the iterative method converges with respect to the number of subdomains. Numerical experiments exemplify the theoretical results.

Axel Målqvist (Chalmers University of Technology)

Numerical simulation of beam network models

Abstract: This presentation considers the numerical solution of Timoshenko beam network models, comprised of Timoshenko beam equations on each edge of the network, which are coupled at the nodes of the network using rigid joint conditions. Through hybridization, we can equivalently reformulate the problem as a symmetric positive definite system of linear equations posed on the network nodes. To discretize the beam network model, we propose a hybridizable discontinuous Galerkin method that can achieve arbitrary orders of convergence under mesh refinement without increasing the size of the global system matrix. As a preconditioner for the typically very poorly conditioned global system matrix, we employ a two-level overlapping additive Schwarz method. We discuss convergence of the corresponding preconditioned conjugate gradient method under appropriate connectivity assumptions on the network.

Florian Schaefer (Courant Institute of Mathematical Sciences, New York University)

Toward Information Geometric Mechanics

Abstract: Shock waves in high-speed gas dynamics cause severe numerical difficulties for classical solvers and scientific machine learning. They are fundamentally a multiscale problem: While viscous effects ensure smoothness on microscopic scales, shocks manifest as macroscopic discontinuities. This talk begins with the observation that shock formation arises from the flow map reaching the boundary of the manifold of diffeomorphisms. We modify its geometry such that geodesics approach but never reach the boundary. The resulting information geometric regularization (IGR) has smooth solutions while avoiding the excessive dissipation of viscous regularizations, accelerating and simplifying the simulation of flows with shocks. We prove the existence of global strong IGR solutions in the unidimensional pressureless case and illustrate its practical utility on multidimensional examples with complex shock interactions. With S. Bryngelson and other collaborators, we use IGR to conduct the first compressible flow simulation exceeding a quadrillion degrees of freedom. The modified geometry of the diffeomorphism manifold is the information geometry of the mass density. The last part of the talk explains how this observation motivates information geometric mechanics that views the solutions of continuum mechanical PDEs as parameters of probability distributions originating from statistical physics. Replacing the Euclidean geometry of individual particles with the information geometry of statistical families promises performant numerical methods that preserve the positivity of densities and energies and readily integrate with scientific machine learning.

Peter Bastian (Heidelberg University)

Practical aspects of some spectral domain decomposition methods

Abstract: Abstract: This presentation will be concerned with the robust and efficient solution of large sparse linear systems arising from discretization of partial differential equations (PDEs) using two-level overlapping domain decomposition preconditioners. To achieve robustness with respect to PDE coefficients, coarse spaces are constructed by solving local eigenproblems. The talk will give an overview of such methods and introduce the GenEO (generalized eigenvalue problems in the overlap) and MS-GFEM (multiscale spectral finite element) approaches. The focus will then be on practical aspects such as choice of subdomains size and various options how to solve the arising eigenvalue problems. The findings will be illustrated by extensive numerical results including also nonsymmetric problems.

Charles Beall (Stevens Institute of Technology)

A randomized Greedy algorithm with certification over the entire parameter set

Abstract: We introduce a randomized Greedy algorithm that can be employed to construct and certify local approximation spaces for linear and nonlinear multiscale PDEs. The algorithm draws random samples of local problem data (such as boundary conditions, source functions, or PDE coefficients) from a clever probability distribution to effectively build a training set at each iteration. We prove that this algorithm provides certification with high probability over the entire parameter set, utilizing results from sampling discretization theory and concentration of measure phenomena. Moreover, we demonstrate favorable properties of the algorithm's sampling complexity which may break the curse of dimensionality encountered by e.g. the deterministic Greedy algorithm when choosing a suitable training set. We present numerical results of the algorithm's performance at building reduced approximation spaces for benchmark PDE problems, and finally investigate the algorithm's construction of local approximation spaces for solutions to the p-Laplace equation, a nonlinear PDE, by sampling local boundary conditions.

Maher Khrais (Bonn University)

Adaptive Iterative Numerical Homogenization for Quasilinear Nonmonotone Elliptic PDE

Abstract: In our talk, we present an adaptive iterative numerical homogenization method in the framework of the localized orthogonal decomposition (LOD). The algorithm is based on a Kačanov iteration, where a new multiscale space construction is performed in each iteration. In our method, we do not update all the correction operators that might be unnecessary if the multiscale solutions slightly change in certain parts of the domain. However, the algorithm adaptively (using local error indicators) recomputes the local corrector problems where it only improves the accuracy of the solution. The motivation is to enhance the efficiency of method when using LOD to solve quasilinear PDE. We also present the well-posedness of the method and the proof of convergence under suitable assumptions on the initial data of the problem. In addition, we illustrate the theoretical findings with numerical experiments, in which we also study a similar algorithm based on Newton iteration, which is only presented experimentally, since its theoretical analysis relies on assumptions that are not guaranteed to hold in our setting.

Malin Mosquera (Chalmers University of Technology)

A Localised Orthogonal Decomposition Method for Heterogeneous Mixed-Dimensional Problems

Abstract: In this talk, we present a model for solving heterogeneous mixed-dimensional elliptic problems. The model is based on the Localised Orthogonal Decomposition (LOD) model and constructs locally supported basis functions on a coarse mesh that are adapted to the problem at hand. The basis ensures optimal convergence with respect to the coarse mesh and leads to an exponentially decaying localisation error. We also present numerical experiments to validate the theoretical findings. Current work entails a similar model to tackle hyperbolic problems.

Daniel Peterseim (University of Augsburg)

Multiscale Random Fields on Quantum Computers

Abstract: We present a quantum algorithm for sampling transformed Gaussian random fields on d-dimensional domains, based on an enhanced version of the classical moving average method. A pointwise nonlinear transformation is used to obtain uniformly bounded and elliptic coefficient fields, as commonly required in multiscale models for partial differential equations. The method generates realizations of the random field directly from a small number of statistical parameters and represents them implicitly as quantum states, rather than storing the full fine-scale realization in classical memory. This circumvents the associated input bottleneck and enables the estimation of linear and nonlinear observables using amplitude estimation techniques. We illustrate the theoretical results with numerical experiments on simulated quantum hardware.

Christian Döding (Bonn University)

Resolving Vortex Lattices in Type-II Superconductors with Multiscale Techniques

Abstract: Superconducting states can be modeled as minimizers of the Ginzburg–Landau energy functional. This functional predicts remarkable macroscopic phenomena, such as the formation of Abrikosov vortex lattices in the presence of an external magnetic field. Accurately computing these states is computationally demanding because classical discretizations require a very fine mesh resolution to capture these characteristics. These requirements can be quantified through error estimates linking the mesh size to the material parameters of the Ginzburg–Landau model. In this talk, I present recent progress toward overcoming these limitations. I propose approximation spaces based on the Localized Orthogonal Decomposition (LOD) method, a multiscale framework that incorporates problem information into computational spaces, substantially relaxing the constraints of classical mesh resolution. Consequently, vortex lattice configurations can be approximated with greater accuracy using fewer degrees of freedom. These advances pave the way for more manageable and reliable computational studies of complex pattern formation in superconductors governed by the Ginzburg–Landau theory.

Dilini Kolombage (University of Bonn)

Subspace Decomposition Preconditioning with Offline–Online Approximation for Random Defect Models

Abstract: We propose an efficient offline-online preconditioning strategy for diffusion problems with random defect coefficients. The method is based on a two-level subspace decomposition framework inspired by [1]. In our method, local patch operators are approximated using a small set of precomputed reference configurations, while the coarse-level operator is assembled exactly in an efficient offline-online manner, following the paradigm introduced for multiscale problems with random defects in [2-3]. Exploiting the localized structure of random defects, all fine-scale patch operators are assembled online as linear combinations of stored offline data, thereby avoiding costly local solves in the online phase and enabling rapid preconditioner assembly for each realization. We establish convergence guarantees by interpreting the offline-online construction as a perturbation of the classical subspace decomposition method. Numerical experiments for random erasure models demonstrate that the proposed approach achieves iteration counts close to those of fully resolved domain decomposition methods at a substantially reduced computational cost, making it particularly well suited for large-scale Monte Carlo simulations. [1]. Kornhuber, Ralf and Yserentant, Harry, Numerical homogenization of elliptic multiscale problems by subspace decomposition, Multiscale Modeling & Simulation. A SIAM Interdisciplinary Journal, 14(3), 1017–1036, 2016. [2]. Målqvist, Axel and Verfürth, Barbara,

An offline-online strategy for multiscale problems with random defects, ESAIM: Mathematical Modelling and Numerical Analysis, 56(1), 237–260, 2022. [3]. Kolombage, Dilini and Verfürth, Barbara, Offline-online approximation of multiscale eigenvalue problems with random defects

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Alexei Lozinski (University of Franche-Comté)

Higher order MsFEM for Stokes equations on a perforated domain

Abstract: We study the Stokes equations in a multiscale setting, modeling viscous flow in perforated domains containing a large number of small obstacles. Our objective is to design numerical methods that deliver accurate approximations on meshes that are coarse relative to both the typical size of the obstacles and their typical separation distance. We present a Multiscale Finite Element Method (MsFEM) for this problem, inspired by the classical Crouzeix–Raviart finite elements [1], and a higher-order generalization, inspired by our recent work on the Localized Orthogonal Decomposition (LOD) [2] for a similar problem. In contrast to LOD, the proposed MsFEM relies on fully local basis functions, with the same compact support as standard finite element basis functions. These basis functions are computed by solving locally the governing equations on each mesh element, with carefully chosen right-hand sides and boundary conditions. The theoretical analysis of the method is based on periodic homogenization theory. [1] G. Jankowiak and A. Lozinski (2024). Non-conforming multiscale finite element method for Stokes flows in heterogeneous media. part II: Error estimates for periodic microstructure. Discrete and Continuous Dynamical Systems-B, 29(5), 2298-2332. [2] M. Hauck and A. Lozinski (2025). A High-Order Localized Orthogonal Decomposition Method for Heterogeneous Stokes Problems. arXiv:2511.22684

Christian Alber (Heidelberg University)

Grassmannian Interpolation of Parametric Local Approximation Spaces

Abstract: Multiscale, parameter-dependent partial differential equations (PDEs) pose severe com putational challenges due to strong coefficient heterogeneity and high-dimensional parameter spaces. We develop a geometric interpolation approach within the multiscale generalized finite element method (MS-GFEM) that targets the most expensive component: computing parameter-dependent optimal local approximation spaces. Leveraging the spatial localization of MS-GFEM and assuming local parameter dependence, we decompose the global problem into parametrically low-dimensional local subproblems. The optimal subspaces for each parameter are identified as points on a Grass- mann manifold and approximated via Grassmann interpolation on sparse grids, which preserves the geometric structure of these spaces while efficiently handling high-dimensional parameter spaces. The resulting localized model reduction method inherits the nearly exponential spatial convergence of MS-GFEM and the parametric convergence rates of sparse grids. Numerical experiments for elliptic problems confirm the theoretical convergence results.

Felix Krumbiegel (Karlsruhe Institute of Technology (KIT))

An enriched higher-order numerical homogenization method for acoustic waves

Abstract: This talk is concerned with the numerical solution of the acoustic wave equation, where the wave propagates through a non-smooth, highly heterogeneous medium. This setup poses many challenges for classical methods as the heterogeneity of the medium needs to be resolved, and high regularity is required for higher-order convergence. Herein, we present a novel tailor-made numerical homogenization strategy that builds on coarse piece-wise polynomials and corrects them accordingly

such that higher-order convergence can be obtained without refining all scales of the medium. In this thesis, the numerical homogenization strategy is first introduced and analyzed in a prototypical setting providing a basis. Further, the method is fully discretized to obtain a computable method. Finally, it is shown that the calculation of these spaces can be performed localized and fully parallelized on small subdomains without a sever impact on the convergence.

Johannes Krauss (University of Duisburg-Essen)

Parameter-robust multilevel block factorization preconditioners based on domain decomposition

Abstract: The first part of this talk provides a brief introduction to multilevel block factorization methods. Referring to the error propagation in inexact two-level methods two commonly used stabilization techniques are discussed, namely, (a) polynomial acceleration, which leads to the class of linear algebraic multilevel iteration (AMLI) methods and (b) Krylov cycles, which result in variable-step preconditioners (nonlinear AMLI). The main convergence results are summarized for both classes of methods. The second part of the talk is intended to illustrate how AMLI and domain decomposition techniques can be combined to construct robust and optimal multilevel preconditioners. A non-variational multigrid algorithm for general symmetric positive definite problems is presented. This method is based on exact two-level block factorization of local (finite element stiffness) matrices that correspond to a sequence of coverings of the domain by overlapping or non-overlapping subdomains. The coarse-grid matrices are defined by means of additive Schur complement approximation and thus the coarse spaces possess a certain energy minimization property. The operator complexity of the multilevel method is controlled by the size and overlap of the subdomains. The two-level method is analyzed in the setting of auxiliary space preconditioning. The last part of the talk addresses several applications. First, it is shown how the proposed framework can be applied to multiscale or highly anisotropic scalar elliptic problems. Next, the construction of uniform preconditioners for highly heterogeneous $H(\mathrm{div})$ problems targeting on the simulation of flow through porous media is discussed. Finally, robust coarse-grid approximations are devised for a family of locking-free discontinuous Galerkin discretizations of the equations of linear elasticity using the same techniques.

Sara Avesani (Università della Svizzera Italiana)

Multiscale scattered data interpolation in samplet coordinates

Abstract: We study multiscale scattered data interpolation schemes for globally supported radial basis functions with focus on the Matérn class. The multiscale approximation is constructed through a sequence of residual corrections, where radial basis functions with different lengthscale parameters are combined to capture varying levels of detail. We prove that the condition numbers of the diagonal blocks of the corresponding multiscale system remain bounded independently of the particular level, allowing us to use an iterative solver with a bounded number of iterations for the numerical solution. We derive a general error estimate bounding the consistency error issuing from a numerical approximation of the multiscale system. To apply the multiscale approach to large data sets, we suggest to represent each level of the multiscale system in samplet coordinates. Samplets are localized, discrete signed measures exhibiting vanishing moments and allow for the sparse approximation of generalized Vandermonde matrices issuing from a vast class of radial basis functions. Given a quasi-uniform set of N data sites, and local approximation spaces with exponentially decreasing dimension, the samplet compressed multiscale system can be assembled with cost $O(N \log^2 N)$. The overall cost of the proposed approach is $O(N \log^2 N)$.
